

**Department of Mathematics**  
**Third Annual**  
**High School Problem Solving Contest**  
**November 1, 2018**

Name: \_\_\_\_\_

School: \_\_\_\_\_

Grade: \_\_\_\_\_

Email: \_\_\_\_\_

1. **10 points**

Two students attempted to solve a quadratic equation,  $x^2 + bx + c = 0$ . Although both students did the work correctly, the first miscopied the middle term and obtained the solution set  $\{-3, 4\}$ . The second student miscopied the constant term and obtained the solution set  $\{-1, 5\}$ . What are the correct solutions?

2. **10 points**

An unbiased coin is tossed. If the result is a head, then a pair of regular unbiased dice is rolled and the number obtained by adding the numbers on the top faces is noted down. If the result is a tail, then a card from a well-shuffled pack of eleven cards numbered  $2, 3, 4, \dots, 11, 12$  is picked and the number on the card is noted down. What is the probability that the noted number is 7 or 8?

3. **10 points**

Prove that for any integer number  $m$ , the value of

$$\frac{m}{3} + \frac{m^2}{2} + \frac{m^3}{6}$$

is also an integer.

4. **10 points**

In  $\triangle ABC$ ,  $AB = 27$ ,  $BC = 29$ , and median  $BM = 26$ . Find the area of  $\triangle ABC$ .

5. **10 points**

There are  $p$  points in space, no four of which are in the same plane with the exception of  $q$  (where  $q < p$ ) points which are all in the same plane. Find the number of (distinct) planes in space each containing three of the points.

6. **10 points**

Does there exist a triangle in the  $xy$ -plane with  $60^\circ$  angle such that its vertices have integer coordinates?