Department of Mathematics
Second Annual
High School Problem Solving Contest
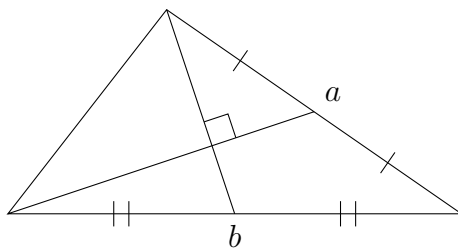# November 8, 2017

Name: _____

School: _____

Grade: _____

Email: _____

1. **10 points**

   Two sides of a triangle are a and b. The medians drawn to these sides intersect at a right angle. Find the third side of the triangle.

2. **10 points**

   For what values of a are both roots of the equation

   $$x^2 + (1 - a)x + a = 0$$

   positive?

3. $\boxed{\textbf{10 points}}$

Each face of a cube is painted red or blue, each with a probability of 1/2. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all of the same color?

4. **10 points**

   Let n be a natural number. An ant wants to crawl a path of length $2n$ along the grid lines that starts and ends at the origin. Prove that there are $\dbinom{2n}{n}^2$ ways to do this, where $\dbinom{m}{k} = \dfrac{m!}{k!(m-k)!}$ is a binomial coefficient.

5. **10 points**

The graph of
$$2x^2 + xy + 3y^2 - 11x - 20y + 40 = 0$$

is an ellipse in the first quadrant of the xy-plane. Let a and b be the minimum and maximum values of y/x, respectively, over all points (x, y) on the ellipse. What is the value of $a + b$?

6. $\boxed{\textbf{10 points}}$

Given that 2^{2017} is a 608-digit number with first digit 1, how many elements of the set $S = \{2^0, 2^1, 2^2, \ldots, 2^{2016}\}$ have a first digit of 4?