Math Field Day 2023

Mad Hatter Marathon 9-10

CSU Fresno Mathematics
www.fresnostate.edu/csm/math/

April 15, 2023

Mad Hatter Marathon 9-10

Welcome to Fresno State!

The Mad Hatter Marathon is a competition in rapid computation and problem solving. You may find that you do not have time to solve every problem. After a few minutes you may feel “mentally out of breath.” Do not let this discourage you. Your fellow contestants feel the same way. That is why this contest is called *Mad Hatter Marathon!*

Mad Hatter 9-10

The Mad Hatter Marathon is divided into two problem solving periods, each lasting 60 minutes. Between the two periods there will be a 15-minute break.

Part I

- This part of the competition consists of 30 problems.
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- You may move to a new question without solving the old one.

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Rules and Scoring

You may use pencil and scratch paper to do calculations, but **calculators are not allowed.**

Your score is the total number of correct answers, so give the best answer that you can in the time available for each problem. There is no penalty for guessing.

Filling Out Your Scantron

- Put your **full name** in the box labeled “Name.”
- Put your **full school name** in the box labeled “Subject.”
- Write “**Part 1**” in the box labeled “Exam.”

Reminders

- Please turn off any devices that could make noise, such as cell phones, beepers, watches, etc.
- If your pencil breaks or needs sharpening, stay in your seat and raise your hand.
- Keep your eyes on your own paper. Keep your Scantron flat on your desk. Contestants caught cheating will be disqualified.

Ready... Set... Go!

Prepare to begin the *Mad Hatter Marathon!*

Part I - Problem 1

Consider a nice, circular analog clock. After how many minutes will the minute hand have moved 15°?

A. 2 minutes
B. $2\frac{1}{2}$ minutes
C. 2 minutes, 45 seconds
D. 3 minutes
E. $3\frac{1}{2}$ minutes

Part I - Problem 2

What is the equation of the perpendicular bisector of the line segment connecting the points $(1, 3)$ and $(-3, 1)$?

A. $y = -\frac{1}{2}x + 2$

B. $y = -\frac{1}{2}x - 2$

C. $y = \frac{1}{2}x$

D. $y = -2x + 2$

E. $y = -2x$

Part I - Problem 3

What is the smallest perfect cube divisible by 12?

A. 64
B. 81
C. 216
D. 8^3
E. 12^3

Part I - Problem 4

If $a < b$, then $3^2 + 4^2 + 5^2 + 12^2 = a^2 + b^2$ can only be satisfied for one pair of numbers (a, b). What is $a + b$?

A. 9
B. 12
C. 17
D. 18
E. 24

Part I - Problem 5

This past Easter, Tom found twice as many eggs as Sam and Sam found three times as many eggs as Maria. If all together, they found 20 eggs, how many eggs did Sam find?

A. 12
B. 6
C. 4
D. 8
E. 2

Part I - Problem 6

For how many distinct integer values of b will the polynomial $x^2 + bx - 16$ have integer roots?

A. 5
B. 6
C. 3
D. 4
E. 2

Part I - Problem 7

The product of p with x^{100} is equal to $(-x)^{100}$. What is p?

A. -100
B. -1
C. 1
D. 100
E. none of the above

Part I - Problem 8

Which of the following is the coefficient of x^2y^3 in the expansion of $(5x + y)^5$?

A. 25
B. 250
C. 350
D. 1250
E. 1350

Part I - Problem 9

$30 + 40 + 50 -$ (the average of 30, 40, and 50) $= \ldots$

A. 0
B. 60
C. 75
D. 80
E. 90

Part I - Problem 10

Find all solutions to the equation $x^3 + 7x^2 - 4x = 28$.

A. $7, 1, 4$
B. $\frac{2}{7}, \frac{3}{7}, \frac{5}{7}$
C. $7, 2, 2$
D. $-7, 2, -2$
E. $7, 2, 3$

Part I - Problem 11

James answered half as many of the questions incorrectly as he did correctly, and he left one-fourth of the questions on the final exam blank. If there were 96 questions on the exam, how many questions did James correctly answer?

A. 48
B. 90
C. 64
D. 24
E. 72

Part I - Problem 12

How many whole numbers have squares that are between 2 and 200, including 2 and 200?

A. 14
B. 11
C. 15
D. 12
E. 13

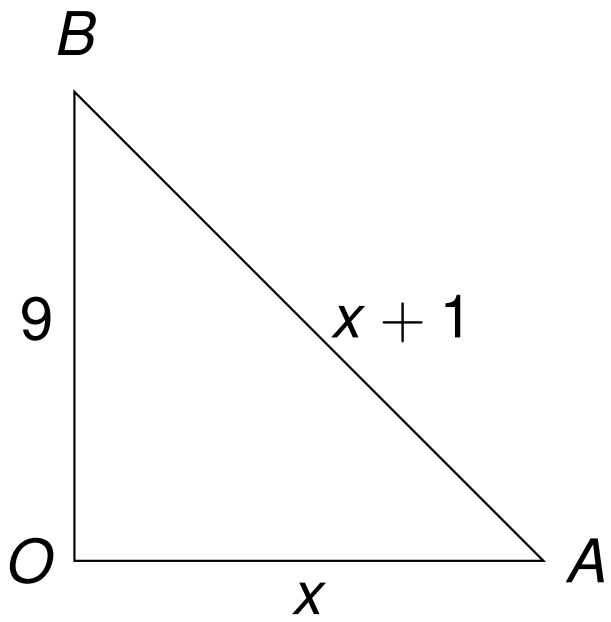
Part I - Problem 13

Including diagonals on the face, what is the total number of diagonals that a cube has?

A. 8
B. 12
C. 14
D. 16
E. 24

Part I - Problem 14

Find the area of right triangle OAB below.



A. 180
B. 360
C. 240
D. 184.5
E. 298

Part I - Problem 15

A puzzle manufacturer makes some of its puzzles more difficult by including extra pieces that do not fit. One puzzle is advertised as having 750 pieces, but the manufacturer has put into the box 5 more pieces that do not fit. What is the probability of opening the box for the first time and pulling out one of the pieces that won't fit?

A. $\frac{1}{150}$
B. $\frac{1}{750}$
C. $\frac{1}{151}$
D. $\frac{1}{5}$
E. $\frac{1}{755}$

Part I - Problem 16

At Bob's Burger Shack, three burgers and one order of fries cost \$4; two burgers and two drinks cost \$7; and three drinks and four orders of fries cost \$9. How much would 4 burgers, 4 drinks and 4 orders of fries cost all together?

A. \$16
B. \$20
C. \$24
D. \$25
E. \$26

Part I - Problem 17

I started with a positive number, and then added 4 to it. Next, I squared the result, then subtracted 4 from that result, then divided that result by 5, and then finally, added 7 to that result. If my final answer was 40, what is the least possible value of the number with which I started?

A. 3
B. 5
C. 7
D. 9
E. 11

Part I - Problem 18

Evaluate $10 + 8 \div 2 \times 4 - 25$.

A. -14
B. 9
C. 0
D. 3
E. 1

Part I - Problem 19

The sum of two integers is 30. The sum of the squares of the same two integers is 500. Which is a quotient of those two integers?

A. 15
B. 2
C. 10
D. 5
E. 12

Part I - Problem 20

Give all values of x for which

$$\frac{x-4}{3x^3-27x}$$

is undefined.

A. $0, \pm 3$
B. $0, \pm 3, 4$
C. $0, 3$
D. $0, -3$
E. ± 3

Part I - Problem 21

Let triangle ABC have vertices at $A(3,4)$, $B(2,5)$, and $C(5,7)$. Which of the following best describes $\triangle ABC$?

A. right scalene triangle
B. right isosceles triangle
C. isosceles triangle
D. acute triangle
E. obtuse triangle

Part I - Problem 22

I have a group of 5 girls and 6 boys. How many ways are there for me to choose 2 girls and 2 boys for a 4-person committee?

A. 150
B. 200
C. 300
D. 450
E. 600

Part I - Problem 23

A school's marching band has fewer than 100 students. When they line up in rows of 4, there is one student left over; when they line up in rows of 5, there are two left over; and when in rows of 6, there are 3 left over. How many students are there in the band?

A. 49
B. 62
C. 81
D. 73
E. 57

Part I - Problem 24

Which of the following is equal to

$$\frac{x^6 y^2 z^{-2}}{x^3 y^3 z^{-4}}?$$

A $\dfrac{x^3}{yz^2}$

B $\dfrac{x^3 y^2}{z^2}$

C $\dfrac{x^3 z^2}{y^2}$

D $\dfrac{x^3 z^2}{y}$

E $\dfrac{x^3 y}{z^2}$

Part I - Problem 25

If $x + y = 11$ and $\sqrt{x} + \sqrt{y} = 4$, what is the value of $\sqrt{xy}$?

A. 6
B. 5
C. $\frac{5}{2}$
D. 3
E. 0

Part I - Problem 26

What is the length, in inches, of the longest stick that will fit into a closed rectangular box whose dimensions are 8 in. $\times$ 9 in. $\times$ 12 in.?

A. 10
B. 12
C. $\sqrt{145}$
D. 15
E. 17

Part I - Problem 27

Isosceles trapezoid $ABCD$ has $AB = 10$, $CD = 20$, $BC = AD$, and its area is 180. What is BC?

A. 10
B. 13
C. 15
D. 17
E. 18

Part I - Problem 28

If a fair coin is flipped five times, what is the probability that it comes up heads three or more times?

A. $\frac{7}{18}$

B. $\frac{5}{12}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

E. $\frac{1}{6}$

Part I - Problem 29

I bought a TV set on sale for $420, which was 40% off its original price. What was its original price?.

A. $1,050
B. $700
C. $672
D. $588
E. $880

Part I - Problem 30

A certain positive number minus its reciprocal is equal to 5/6. What is this number?

A. $\frac{2}{3}$

B. $\frac{1}{6}$

C. $\frac{5}{12}$

D. $\frac{3}{2}$

E. $\frac{12}{5}$

Mad Hatter - 15-minute break

Whew! You've reached the end of Part I.

- Please make sure your full name and school name are on your Scantron form.
- Pass your Scantrons in.
- You may leave your belongings here during the break.
- Part II will begin promptly in **15 minutes**.

Mad Hatter - Part II

The rules for this part of the competition are the same as the previous part.

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Filling Out Your Scantron

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- Write “**Part 2**” in the box labeled “Exam.”

Ready... Set... Go!

Prepare to restart the *Mad Hatter Marathon!*

Part II - Problem 1

There is a double feature showing at the movies. The first film begins at 5:25 PM and lasts one hour and fifty minutes. Then there is a twenty minute intermission. The second film lasts two hours and forty minutes. What time does the second film end?

A. 10:25 PM
B. 10:20 PM
C. 10:15 PM
D. 10:10 PM
E. 10:05 PM

Part II - Problem 2

At a certain time of day, a 6-foot post casts a 4-foot shadow. At the same time a flagpole casts a 28-foot shadow. How many feet high is the flagpole?

A. $\frac{56}{3}$ feet
B. 42 feet
C. 76 feet
D. 56 feet
E. 72 feet

Part II - Problem 3

What is the last digit of the number 2023^{2023}?

A. 0
B. 3
C. 4
D. 6
E. 7

Part II - Problem 4

There are a total of 81 students taking a foreign language in a high school where they offer Spanish, French, and German. There are 21 students who take at least two languages at once. If 55 students are taking Spanish, 28 students taking French, and 24 students taking German, how many students are taking all three languages at once?

A. 25
B. 13
C. 11
D. 8
E. 5

Part II - Problem 5

In a circle, draw chords $\overline{AB}$ and $\overline{CD}$ so that they intersect inside the circle at E. Also draw chords $\overline{AC}$ and $\overline{BD}$. If the degree measure of angle CAE is 80, and the degree measure of angle DBE is 65, what is the degree measure of angle AED?

A. 95
B. 45
C. 100
D. 145
E. 115

Part II - Problem 6

A sphere is inscribed in a cube. What is the exact value of the ratio of the volume of the sphere to the volume of the cube?

A $\frac{\pi}{12}$

B $\frac{\pi}{8}$

C $\frac{\pi}{6}$

D $\frac{\pi}{4}$

E $\frac{\pi}{3}$

Part II - Problem 7

Jack sets out on a bike ride at 18 miles per hour. Jill sets out 10 minutes later and follows the same route at 22 miles per hour. How many minutes after she starts will Jill catch up to Jack?

A. 15
B. 30
C. 45
D. 55
E. 60

Part II - Problem 8

If $f\left(\frac{x+3}{x}\right) = x^2 + 6x + 2$, what is $f(-1)$?

A. -4

B. $-\frac{19}{4}$

C. 9

D. $-\frac{3}{4}$

E. -3

Part II - Problem 9

A water tank is made in the shape of a horizontal circular cylinder with hemispherical caps at both ends. The length of the tank, including the caps, is 16 feet and the height is 6 feet. What is the volume of this tank?

A. 90π cubic feet

B. $\frac{189}{2}\pi$ cubic feet

C. 108π cubic feet

D. 126π cubic feet

E. 99π cubic feet

Part II - Problem 10

Four consecutive odd integers add up to 464. What is the largest of these integers?

A. 111

B. 113

C. 115

D. 117

E. 119

Part II - Problem 11

The pages of a certain book are numbered consecutively from 1 to 250. How many times does the digit 7 appear among the page numbers?

A. 45

B. 50

C. 52

D. 61

E. 19

Part II - Problem 12

Sterling silver is 92.5% pure silver. How many grams of sterling silver must be mixed to a 90% silver alloy to obtain 500 grams of a 91% silver alloy?

A. 100
B. 140
C. 200
D. 300
E. 360

Part II - Problem 13

Two cubes with sides of integer lengths have their combined volume equal to the sum of the lengths of all their edges. What is their combined volume?

A. 91
B. 72
C. 35
D. 152
E. 65

Part II - Problem 14

Tom can whitewash a fence in 2 hours, and Huck can do the same job in 3 hours. If Tom, working alone, gets one-third of the fence whitewashed, how many additional hours will it take Tom and Huck, working together, to finish the job?

- (A) $\frac{4}{5}$ hour
- (B) $\frac{3}{4}$ hour
- (C) $\frac{2}{3}$ hour
- (D) $\frac{1}{2}$ hour
- (E) 1 hour

Part II - Problem 15

A rectangle, twice as wide as it is high, is inscribed in a circle. What is the ratio of the area of the rectangle to the area of the circle?

- A $\frac{6}{5\pi}$
- B $\frac{4}{3\pi}$
- C $\frac{9}{4\pi}$
- D $\frac{16}{9\pi}$
- E $\frac{8}{5\pi}$

Part II - Problem 16

Suppose that L is a list of positive integers, not necessarily distinct, and that the number 80 is present. The average of this set is 66. When 80 is removed, the average drops to 65. What is the largest possible number in L?

A. 140
B. 460
C. 844
D. 897
E. 910

Part II - Problem 17

What is one fifth of one half of two thirds of 45?

A. $\frac{1}{2}$
B. 1
C. 2
D. 3
E. 4

Part II - Problem 18

What is the smallest positive integer n such that

$$\sqrt{n} - \sqrt{n-1} < \frac{1}{6}?$$

A. 25
B. 16
C. 9
D. 4
E. 1

Part II - Problem 19

Suppose the first five terms of an arithmetic progression are $a, x, b, 2x, c$. What is the ratio of c to a?

A. 5

B. 4

C. $\frac{5}{2}$

D. $\frac{5}{3}$

E. $\frac{5}{4}$

Part II - Problem 20

Let a and b be integers whose sum is evenly divisible by 3. Which of the following must be true?

I. $a^2 + b^2$ is evenly divisible by 3

II. $a^2 - b^2$ is evenly divisible by 3

III. $a^3 + b^3$ is evenly divisible by 3

A. I only

B. II only

C. I and III

D. II and III

E. I, II, and III

Part II - Problem 21

Suppose the triangle ABC has a right angle at C and that $BC = 3$ and $AC = 4$. What is the length of the angle bisector at B?

A. $2\sqrt{3}$

B. $\frac{7}{2}$

C. $\frac{3}{2}\sqrt{5}$

D. $3\sqrt{2}$

E. 4

Part II - Problem 22

Huey, Dewey, and Louie are making pizzas. Huey's is a 10-inch by 10-inch square. Dewey's is a circle of diameter 12 inches. Louie's is a 6-inch by 24-inch rectangle. Arrange the pizzas according to size:

A. Huey's < Dewey's < Louie's
B. Dewey's < Huey's < Louie's
C. Dewey's < Louie's < Huey's
D. Huey's < Louie's < Dewey's
E. Louie's < Dewey's < Huey's

Part II - Problem 23

Jorge's favorite number is 5, so he decides to build a tower 555 inches high. He has blocks of height 30 inches and blocks of height 21 inches. What is the smallest number of blocks that he can use?

A. 20
B. 22
C. 24
D. 26
E. 28

Part II - Problem 24

There are seven players participating in a ping-pong tournament. Each player will play every other player three times. How many games will be played altogether?

A. 147
B. 126
C. 108
D. 90
E. 63

Part II - Problem 25

If two standard dice are rolled, what is the probability that the sum of the numbers showing is six or less?

- A $\frac{1}{2}$
- B $\frac{5}{12}$
- C $\frac{1}{3}$
- D $\frac{1}{4}$
- E $\frac{7}{12}$

Part II - Problem 26

What is the area of the triangle formed by the lines $x = 2$, $y = 3$, and $2y + 3x = 18$ in the xy-plane?

A. $\frac{3}{2}$

B. 2

C. $\frac{5}{2}$

D. 3

E. 6

Part II - Problem 27

What is the value of x if

$$6^{x+1} - 6^x = 1080?$$

A. 2
B. 3
C. 4
D. 5
E. 6

Part II - Problem 28

Regular gas costs \$4 per gallon and premium gas costs \$4.20 per gallon. If a car gets 20 mpg using regular gas, by how many mpg must the car's fuel efficiency improve for the premium gas to be equally cost effective?

A. 1 mpg
B. 2 mpg
C. 3 mpg
D. 4 mpg
E. 5 mpt

Part II - Problem 29

Dylan takes AP Calculus BC as a freshman. She wants to maintain a 95% average on all of her exams. Her scores (as percentages) for her first 5 exams were 84, 92, 100, 89, 100. Her teacher offers extra credit points on the sixth and final exam (so, points awarded beyond earned score). Assuming Dylan earns a 100 on the final exam, how many extra percentage points did her teacher award Dylan in order for her to maintain her 95% average? (Note: Only whole numbers of points are awarded.)

A. 2
B. 5
C. 8
D. 12
E. 14

Part II - Problem 30

How many cubic feet of topsoil do I need to elevate a 12 foot by 12 foot rectangular garden by 4 inches.

A. 576 cubic feet
B. 432 cubic feet
C. 192 cubic feet
D. 96 cubic feet
E. 48 cubic feet

Mad Hatter - Done!

You made it!

- Please make sure your full name and school name are on your Scantron form.
- Pass your Scantron in.
- Please take your belongings with you.
- There will be games and other fun activities in Science II, Room 308, from 1:00-2:30pm.
- The awards ceremony will begin at **2:45pm**. If there are any ties, you have to be present to win the tiebreaker. See you there!

Part I

1	B	6	A	11	A	16	A	21	C	26	E
2	E	7	C	12	E	17	D	22	A	27	B
3	C	8	B	13	D	18	E	23	E	28	C
4	D	9	D	14	A	19	B	24	D	29	B
5	B	10	D	15	C	20	A	25	C	30	D

Part II

1	C	6	C	11	A	16	D	21	C	26	D
2	B	7	C	12	C	17	D	22	A	27	B
3	E	8	B	13	B	18	B	23	A	28	A
4	E	9	D	14	A	19	A	24	E	29	B
5	D	10	E	15	E	20	D	25	B	30	E