No calculators allowed

Correct Answer = 4, Incorrect Answer = −1, Blank = 0

1. Points P and R are located at $(2, 1)$ and $(12, 15)$, respectively. Point M is at the midpoint of segment $\overline{PR}$. Segment $\overline{PR}$ is reflected over the x-axis. What is the sum of the coordinates of the image of point M (the midpoint of the reflected segment)?

   (a) 2   (d) 1

   (b) −2   (e) −1

   (c) 0

2. The difference between the squares of two consecutive natural numbers is 27 more than the larger of the two natural numbers. What is the product of the two natural numbers?

   (a) 784   (d) 792

   (b) 730   (e) 720

   (c) 756

3. The numbers 1 to 8 are placed at the vertices of a cube in such a manner that the sum of the four numbers on each face is the same. What is this common sum?

   (a) 12   (d) 36

   (b) 18   (e) 48

   (c) 24

4. Define $a@b = ab - b^2$ and $a\#b = a + b - ab^2$. What is $3@(6\#2)$?

   (a) -304

   (b) -296

   (c) -48

   (d) 36

   (e) None of the above

5. Of the 500 balls in a large bag, 80% are green and the rest are brown. How many of the green balls must be removed from the bag so that 75% of the remaining balls are green?

   (a) 25

   (b) 125

   (c) 75

   (d) 100

   (e) 50

6. Find the sum of all the distinct primes (without repetition) in the prime factorization of 2023.

   (a) 24

   (b) 36

   (c) 42

   (d) 30

   (e) 48

7. Julia can ride her bicycle uphill at 5 miles per hour and downhill at 15 miles per hour. How far uphill in miles should she travel if she wants her round trip (uphill plus downhill) to last exactly half an hour?

   (a) 1.525 miles

   (b) 1.675 miles

   (c) 1.875 miles

   (d) 2.325 miles

   (e) 2.375 miles

8. Suppose that the four-digit number $74p8$ is divisible by 12. Find the sum of all possible values of the digit p.

(a) 15

(b) 10

(c) 7

(d) 20

(e) 12

9. Jose has 9 blue socks, 18 black socks, and 10 white socks all mixed up in his drawer. It's late at night, and he doesn't want to turn on the light in the dark room. What is the smallest number of socks he has to pull out of his drawer to guarantee that he will have a pair of black socks?

(a) 17

(b) 18

(c) 19

(d) 20

(e) 21

10. Todd's friend Olivia is flying her plane at a constant elevation of 1.5 km. From the ground, Todd sees the plane moving in his direction from the west at a 30 degree angle of elevation. One minute later, after Olivia had flown directly overhead, Todd turns and sees the plane moving away from him to the east at a 45 degree angle of elevation. How fast is Olivia flying in kilometers per hour?

(a) About 230

(b) About 250

(c) About 270

(d) About 290

(e) About 310

No calculators allowed

Correct Answer = 4, Incorrect Answer = −1, Blank = 0

11. Sets A, B, and C all contain positive whole numbers that are less than 30 according to the definitions below:
    Set $A = \{$multiples of 4$\}$
    Set $B = \{$numbers that are 1 less than a prime$\}$
    Set $C = \{$multiples of 3$\}$
    Find the sum of the elements in the set $(A \cap C) \cup (B \cap C)$.

    (a) 60

    (b) 72

    (c) 4

    (d) 12

    (e) 48

12. When Demetri bought his car, he paid $1/4$ of the total price right away. To pay off the rest, each month he pays $1/5$ of the original price as a payment. How many monthly payments will it take Demetri to pay off his car? *Note that the final payment may be less than $1/5$ of the original price.*

    (a) 1

    (b) 2

    (c) 3

    (d) 4

    (e) 5

13. Karla went to the store and bought three dozen eggs for $4.80 per dozen. On her way home she met Tracy and gave Tracy back the $6.00 she borrowed from her last week. She now has exactly half the money she had before going to the store. How much money does Karla have now?

    (a) $20.40

    (b) $14.40

    (c) $40.80

    (d) $18.00

    (e) $8.40

14. When each side of a square was increased in length by 50%, its area increased by 180 square inches. How many square inches are in the original square?

(a) 240

(d) 90

(b) 144

(e) 80

(c) 100

15. The ordered list of numbers $18, 21, 24, 26, A, 36, 37, B$ has a median of 30 and a mean of 30. Find $B - A$.

(a) 0

(d) 11

(b) 5

(e) None of the above

(c) 10

16. The sum of the digits of 2023 is $2+0+2+3 = 7$ and 7 is a factor of 2023. For how many numbers in the 2020's is the sum of the digits a factor of the number?

(a) 5

(d) 4

(b) 3

(e) 6

(c) 2

17. The product of the ages of three teenagers is 4590. How old is the oldest?

(a) 14

(d) 17

(b) 15

(e) 18

(c) 19

18. Pat averages 12 MPH riding their bicycle to school. Averaging 36 MPH by car takes them one-half hour less time. How far do they travel to school?

(a) 15 miles

(d) 9 miles

(b) 20 miles

(e) 36 miles

(c) 12 miles

19. The value of $3^3 = 27$. The *units digit* (ones place) for 3^3 is 7. What is the units digit for 3^{122}?
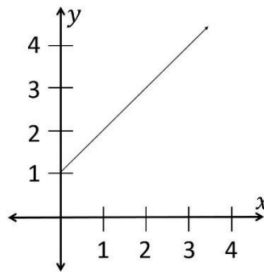
(a) 1

(d) 7

(b) 3

(e) 4

(c) 9

20. The following functions f, g, and h represent the distance, y, traveled at time x by three different objects (measured in the same units).

f:

x	y
0	1
5	6
10	11

g: $x = \frac{y}{4} - \frac{3}{8}$

h:

Which object is moving fastest?

(a) The object represented by function f.

(b) The object represented by function g.

(c) The object represented by function h.

(d) The objects represented by the functions f and h, which are traveling at the same speed.

(e) The answer cannot be determined with the information provided.