

No calculators allowed

Correct Answer = 4, Incorrect Answer = -1, Blank = 0

1. How many (distinct) real number solutions are there to the equation

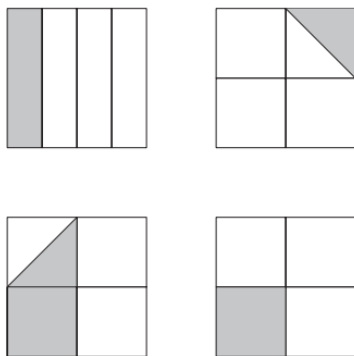
$$x^{2022} - 4x^{2020} = x^{2020} - 4x^{2018} ?$$

- (a) 2
 (b) 3
 (c) 4
 (d) 5
 (e) None of these

2. A palindromic number is a number that is exactly the same if the order is reversed; for example, 1225221 or 1331. How many four-digit palindromic numbers are there for which the sum of its digits is divisible by four?

- (a) 20
 (b) 45
 (c) 50
 (d) 100
 (e) None of these.

3. Each of the following four squares are the same size. Each square is subdivided into congruent rectangles or congruent triangles. What percentage of the total area is shaded?



- (a) 12.5
 (b) 20
 (c) 25
 (d) $33\frac{1}{3}$
 (e) 37.5

4. If $3^p + 3^4 = 90$, $2^r + 44 = 76$, and $5^3 + 6^s = 1421$, what is the product of p , r , and s ?
- (a) 27 (d) 70
(b) 40 (e) 90
(c) 50
5. Jamal drives from his apartment to the airport to catch a flight. He drives 35 miles in the first hour, but realizes he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?
- (a) 149 (d) 210
(b) 175 (e) 245
(c) 180
6. Sarah buys 4 muffins and 3 bananas. Chris spends twice as much on 2 muffins and 16 bananas. A muffin is how many times more expensive than a banana?
- (a) $\frac{3}{2}$ (d) 2
(b) $\frac{5}{3}$ (e) $\frac{13}{4}$
(c) $\frac{7}{4}$
7. A parabola $y = ax^2 + bx + c$ has its vertex at $(6, 15)$ and contains the point $(0, -3)$. What is the product abc ?
- (a) $-\frac{4}{9}$ (d) 9
(b) -1 (e) None of these
(c) 4

8. What is the value of $4 * (-1 + 2 - 3 + 4 - 5 + 6 - \dots - 999 + 1000)$?

(a) -10

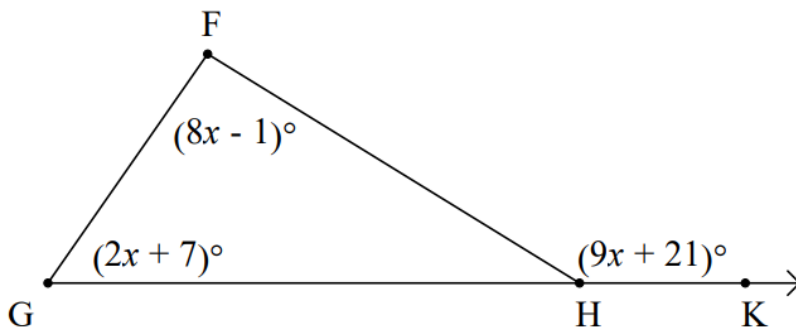
(d) 500

(b) 0

(e) 2000

(c) 1

9. What is the measure of the angle $\angle FGH$?



(a) 15°

(d) 57°

(b) 22°

(e) 119°

(c) 37°

10. If $9^{5x+3} = 27^{2x+1}$, then $81^x = ?$

(a) $-\frac{3}{4}$

(d) $\frac{1}{3}$

(b) $\frac{1}{27}$

(e) None of These

(c) $\frac{1}{9}$

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11. Suppose

$$\frac{x^3 + x^2 + cx + d}{x + 2} = ax^2 + bx + 4$$

for all x except $x = -2$. What is $3^2a + 3b + 4$?

(a) $\frac{38}{5}$

(d) $\frac{54}{5}$

(b) 9

(e) None of the above

(c) 10

12. The lines $y = 2x + b$ and $y = x + 2022$ meet at a point on the line $y = 4x + 18$. Determine b .

(a) 1250

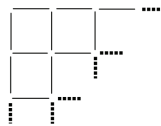
(d) 1350

(b) 1251

(e) None of the above

(c) 1351

13. A grid is formed with toothpicks that is 60 toothpicks across and 32 toothpicks high. How many toothpicks are needed to form this grid?



(a) 1920

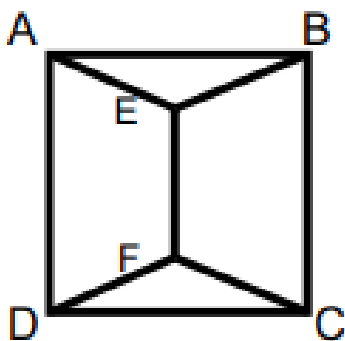
(d) 1980

(b) 2952

(e) 3932

(c) 2022

14. If cars hold 5 passengers and charge \$29 for a trip to the airport and vans hold 7 passengers and charge \$41 for a trip to the airport, find the minimum cost to transport 49 people to the airport.
- (a) \$280 (d) \$290
 (b) \$285 (e) None of the above
 (c) \$287
15. Define the operation $\$$ by $a\$b = a + b - ab$. So for example $7\$10 = 7 + 10 - 70 = -53$. What is $(2\$3)\4 ?
- (a) 4 (d) 7
 (b) 5 (e) None of the above
 (c) 6
16. A fair coin is tossed 3 times. What is the probability of at least two consecutive heads?
- (a) $\frac{1}{8}$ (d) $\frac{1}{2}$
 (b) $\frac{1}{4}$ (e) $\frac{3}{4}$
 (c) $\frac{3}{8}$
17. Suppose the square ABCD has side length 6. Suppose all interior segments AE , BE , DF , CF , and EF have the same length. Find the length of EF .



- (a) $\frac{6 + \sqrt{15}}{2}$ (d) $2(\sqrt{2} + \sqrt{3})$
 (b) $2\sqrt{7} - 2$ (e) None of the above
 (c) $\frac{79}{24}$

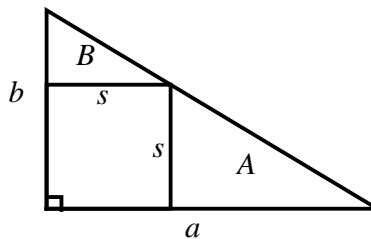
18. At the start of class all thirty students are awake. During the class, students fall asleep at a rate of one student every thirty seconds. Ten minutes into the class, a sleeping student wakes up. The sleeping students continue to wake up at a rate of one student every minute. Any student who woke up can fall back asleep. How many minutes to the first time in class that all students will be asleep?

- (a) 19.5 (d) 25.5
 (b) 20.5 (e) 30.5
 (c) 22.5

19. What is the ones digit for the product $(5 + 1)(5^3 + 3)(5^6 + 6)(5^{12} + 12)$?

- (a) 0 (d) 6
 (b) 2 (e) 8
 (c) 4

20. In the figure below, the large right triangle has respective leg lengths a and b , as pictured. The s by s square is inscribed in the triangle. The respective areas of the two smaller right triangles are A and B as indicated. Determine the ratio of the areas A/B as a function of a and b .



- (a) $A/B = a^2/b^2$ (d) $A/B = \sqrt{a^2 + b^2}/(a + b)$
 (b) $A/B = a/b$ (e) None of the above
 (c) $A/B = (ab)/(a + b)$