2015
Leap Frog Relay Grades 11-12
Part I

No calculators allowed
Correct Answer = 4, Incorrect Answer = -1, Blank = 0

1. How many positive factors does 2015^4 have?

   (a) 4 (b) 12

   (c) 64 (d) 125

   (e) None of these

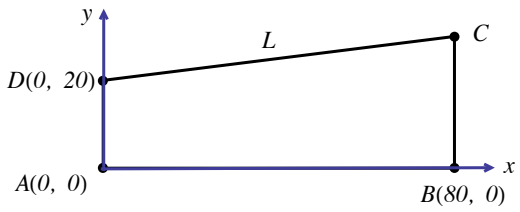
2. Kate drew blue and red rectangles on a blackboard. Exactly 7 of the rectangles are squares. There are 3 more red rectangles than blue squares. There are 2 more red squares than blue rectangles. How many blue rectangles are there on the blackboard?

   (a) 1 (b) 3

   (c) 5 (d) 6

   (e) None of these

3. What is the slope of the line L if the area enclosed by the trapezoid $ABCD$ is equal to 2015?



(a) Slope of $L = \dfrac{83}{640}$

(b) Slope of $L = \dfrac{83}{720}$

(c) Slope of $L = \dfrac{93}{640}$

(d) Slope of $L = \dfrac{93}{720}$

(e) None of these

4. Suppose that when dividing the number n by 7, there results a remainder of 3. What then is the remainder if you were to divide the number $2015n$ by 7?

(a) 0

(b) 1

(c) 2

(d) 3

(e) None of these

5. If the vertex of the parabola $y = ax^2 + bx + c$ lies on the x-axis, then

(a) $b^2 = 4ac$

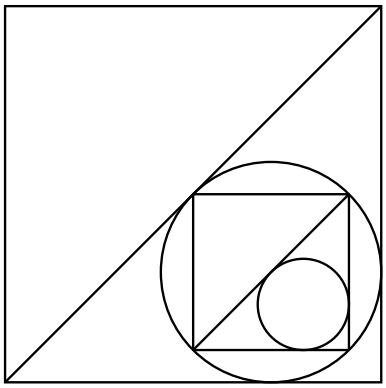
(b) $c = 0$

(c) $a + b + c = 0$

(d) $b = 0$

(e) None of these

6. In the figure below, the big square has side length a and its diagonal is drawn. A circle is inscribed in one of the two obtained triangles as pictured. A square is inscribed in that circle, and then the procedure is repeated. Determine the radius, r, of the smaller circle.



(a) $r = a(\sqrt{2}/2)$

(b) $r = a(1 - \sqrt{2}/2)$

(c) $r = a(\sqrt{2} - 1)$

(d) $r = a(3\sqrt{2}/2 - 2)$

(e) None of these

7. *For how many* of the ten digits $x = 0, 1, 2, \ldots, 9$ is the 2017-digit number $n = 1\underbrace{xx\ldots x}_{2015}0$ divisible by 24?

(a) 0

(b) 1

(c) 2

(d) 3

(e) None of these

8. How many regular polygons exist such that the measure of each one of their angles in degrees is an integer?

(a) 17

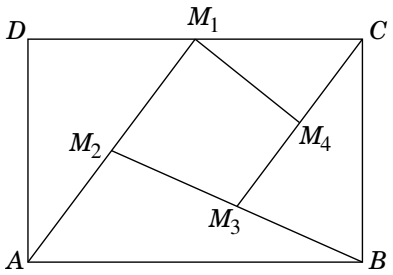
(b) 18

(c) 22

(d) 25

(e) None of these

9. In the rectangle $ABCD$ shown in the figure, M_1 is the midpoint of DC, M_2 is the midpoint of AM_1, M_3 is the midpoint of BM_2 and M_4 is the midpoint of CM_3. Find the ratio between the areas of the quadrilateral $M_1M_2M_3M_4$ and of the rectangle $ABCD$.



(a) $\frac{7}{32}$ (b) $\frac{3}{16}$

(c) $\frac{1}{5}$ (d) $\frac{2}{9}$

(e) None of these

10. Let $x = \frac{1}{2}(\sin^{-1}(3/5) + \sin^{-1}(5/13))$. What is the value of $\tan x$?

(a) 7/12 (b) 5/8

(c) 4/7 (d) 6/13

(e) None of these

Correct answers: 1 (d) 2 (b) 3 (a) 4 (e) (the reminader is 4) 5 (a) 6 (d) 7 (b) 8 (c) 9 (a) 10 (c)

2015
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Part II

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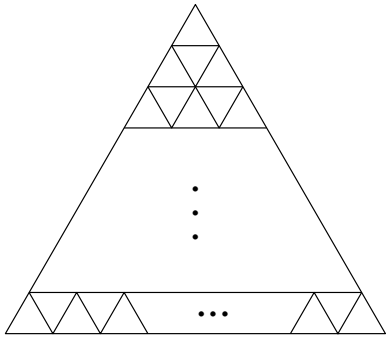
11. Sticks are placed on a table to form a big triangle consisting of smaller triangles as pictured below. Assuming each small triangle side consists of a single stick and each big triangle side consists of 2015 sticks, how many sticks are used in all?



(a) 2,031,120

(b) 4,058,210

(c) 6,093,360

(d) 8,120,450

(e) None of these

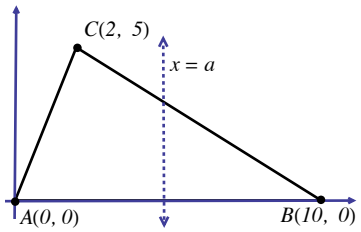
12. How many 3-digit positive integers can be represented as the sum of exactly nine different powers of 2?

(a) 0 (b) 1

(c) 3 (d) 5

(e) None of these

13. What is the value of a so that the vertical line $x = a$ divides the triangle $\triangle ABC$ pictured below into two regions of equal area?



(a) $a = \sqrt{7}$ (b) $a = \dfrac{7}{2}$

(c) $a = 3$ (d) $a = 5 - \sqrt{5}$

(e) None of these

14. How many integers between 1 and 2015 have exactly 27 positive divisors?

(a) 0 (b) 1

(c) 2 (d) 3

(e) None of these

15. What is the equation of the line with positive slope that goes through the origin and is tangent to the circle $(x-4)^2+y^2=4$?

(a) $y = x/\sqrt{11}$ (b) $y = x/\sqrt{7}$

(c) $y = x/\sqrt{5}$ (d) $y = x/\sqrt{3}$

(e) None of these

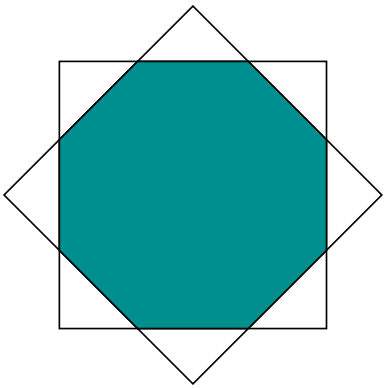
16. How many polynomials $p(x)$ satisfy both $p(12) = 12!$ and $xp(x-1) = (x-12)p(x)$?

(a) 0 (b) 1

(c) 2 (d) infinitely many

(e) None of these

17. Two $2' \times 2'$ squares share the same center and one square is rotated $45°$ with respect to the other square (see picture below). Determine the shaded area that is enclosed by both squares.



(a) Shaded Area $= 4\sqrt{2} - 4$ ft^2. (b) Shaded Area $= 4\sqrt{2} + 4$ ft^2.

(c) Shaded Area $= 2\sqrt{2} + 2$ ft^2. (d) Shaded Area $= 8\sqrt{2} - 8$ ft^2.

(e) None of these

18. What is the radius of the inscribed circle of a triangle with sides 5, 6, and 7?

(a) radius $= \frac{2\sqrt{5}}{3}$

(b) radius $= \frac{3}{2}$

(c) radius $= \sqrt{3}$

(d) radius $= \frac{\sqrt{10}}{2}$

(e) None of these

19. The number $\sqrt{20+\sqrt{15}}$ is the root of a degree 4 polynomial $p(x) = x^4 + bx^3 + cx^2 + dx + e$ with integer coefficients. That is, b, c, d and e are all integers. Determine the value of $p(1)$.

(a) 346

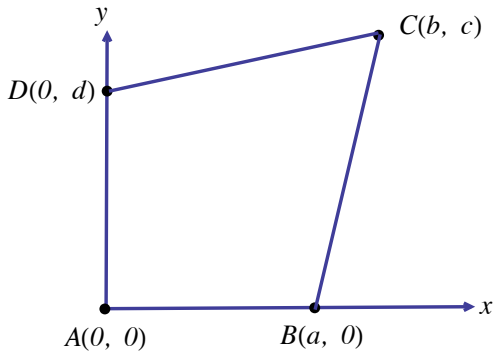
(b) 348

(c) 350

(d) 352

(e) None of these

20. Quadrilateral $ABCD$ in the cartesian plane is pictured below. Determine the area enclosed by $ABCD$. (You may assume $b > a$ and $c > d$ as pictured.)



(a) Area $= \frac{1}{4}(a+b)(d+c)$

(b) Area $= \frac{1}{4}(a+d)(b+c)$

(c) Area $= \frac{1}{2}(ad+bc)$

(d) Area $= \frac{1}{2}(ac+bd)$

(e) None of these

Correct answers: 11 (c) 12 (d) 13 (e) $(10 - 2\sqrt{10})$ 14 (c) 15 (d) 16 (b) 17 (d) 18 (e) $(2\sqrt{6}/3)$ 19 (a) 20 (d)