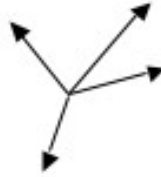# 2009
Leap Frog Relay Grades 6-8
Part I Solutions

No calculators allowed
Correct answer $= 4$, Incorrect answer $= -1$, Blank $= 0$

1. How many angles are there in the figure?

   (a) 4

   (b) 6

   (c) 7

   (d) 8

   (e) More than 8.

   Solution. (e) More than 8.

2. Three watermelons and two cantaloupes weigh 32 pounds. Four watermelons and three cantaloupes weigh 44 pounds. All watermelons weigh the same and all cantaloupes weigh the same. What is the weight of two watermelons and one cantaloupe?

   (a) 15 pounds

   (b) 30 pounds

   (c) 11 pounds

   (d) 20 pounds

   (e) 25 pounds

   Solution. (d) Let w be the weight of a watermelon and let c be the weight of a cantaloupe. The given data can be put into the following system of equations:

   $3w + 2c = 32$ and $4w + 3c = 44$. Subtracting the first equation from the second equation we obtain $w + c = 12$. Solving for c and plugging it into the first equation we get $3w + 2(12w) = 32$, thus $w = 32 - 24 = 8$. So watermelons are 8 pounds and cantaloupes are 4 pounds. We conclude that 2 watermelons and 1 cantaloupe weight 20 pounds.

3. Convert the number 1.23434.... into a *reduced* fraction of integers.

  (a) $\dfrac{3}{2}$

  (b) $\dfrac{611}{495}$

  (c) $\dfrac{16}{11}$

  (d) $\dfrac{12343434}{100000}$

  (e) $\dfrac{1222}{990}$

Solution. **(b)** Let $X = 1.23434$, then $10X = 12.3434$. Now take $100(10X) = 1000X = 1234.3434$ and subtract $10X = 12.3434$ to get: $1000X - 10X = 990X = (1234 - 12) = 1222$. Solving for X, we obtain $X = \dfrac{1222}{990} = \dfrac{611}{495}$.

4. A team of scientists found that there were 4 oak trees for every 10 pine trees. How many oak trees were there if they counted 36 more pine than oak?

  (a) 24

  (b) 30

  (c) 22

  (d) 26

  (e) 36

Solution. (a) Since $\dfrac{4}{10} = \dfrac{8}{20} = \dfrac{12}{30} = \dfrac{16}{40} = \dfrac{20}{50} = \dfrac{24}{60}$ and $60 - 24 = 36$, there are 24 oak trees.

5. Express the next term in the arithmetic sequence $2_3, 13_4, 22_5, 25_6, \ldots$

  (a) 27_7

  (b) 54_7

  (c) 23_7

  (d) 31_7

  (e) 32_7

Solution. (d) Converting the numbers to a common base 10, we have: $2, 7, 12, 17, \ldots$ which tells us that the sequence is arithmetic, adding 5 each time. The next number in base ten is thus 22. Finally, 22 in base 7 is: $3(7^1) + 1(7^0) = 31_7$.

6. A picture that measures 12 cm by 18 cm is enlarged to 4 times its area. What are the new dimensions?

   (a) 48 cm by 72 cm

   (b) 16 cm by 22 cm

   (c) 24 cm by 36 cm

   (d) 122 cm by 168 cm

   (e) 3 cm by 6 cm

   Solution. (c) To get 4 times the area you need to double each dimension.

7. Mrs. O. Angle has 12 students in her advanced algebra class. On a recent test, the average was 80. When Beth asked for her score, Mrs. Angle could not find it, but she did remember adding Beth's score in the average. She also remembered that Beth had passed. These are the scores of the other students:

   $$65,\ 65,\ 70,\ 70,\ 80,\ 85,\ 85,\ 90,\ 90,\ 90,\ 100$$

   What was Beth's score?

   (a) 65

   (b) 70

   (c) 80

   (d) 85

   (e) 90

   Solution. (b) The sum of the scores of the other students is 890. Let x be Beth's score. Then we have $\dfrac{x + 890}{12} = 80$. Solving for x we obtain $x = 70$.

8. Each of the squares shown is inscribed in a larger square so that the vertices of the inscribed square bisect the sides of the larger square. What fraction of the area of the largest square is shaded? Express your answer in simplest form.
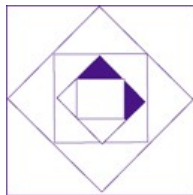
   (a) $\dfrac{1}{24}$

   (b) $\dfrac{1}{32}$

   (c) $\dfrac{1}{2}$

(d) $\dfrac{1}{12}$

(e) $\dfrac{1}{48}$

Solution. (b) Note that the area of each inscribed square is half of the area of the larger square. Consequently, the area of the smallest square in the picture is $\frac{1}{16}$ of the area of the largest square. Finally, the area of the shaded region is half of the area of the smallest square, and thus its area represents $\frac{1}{32}$ of the area of the largest square.

9. Assume that each letter F, I, M, N, S, U, W represents a digit from $1, 2, 3, 6, 7, 9, 0$. Find the numerical value of the sum (SWIM) if you know that

$$
\begin{array}{ccccc}
 & S & U & N \\
+ & F & U & N \\
\hline
S & W & I & M
\end{array}
$$

(a) 1072

(b) 2379

(c) 3211

(d) 7903

(e) 1723

Solution. (a) $1 = S, 2 = M, 3 = U, 6 = N, 7 = I, 9 = F, 0 = W$.

Thus $136 + 936 = 1072$.

10. Shawn bought a car for \$5600. He sold it to Rachael for 5/6 the price he paid for it. Rachael sold it to Ray for 1/5 less than she paid for it. Ray sold it to Rick for 3/4 what he paid. What did Rick pay for the car?

(a) \$1600

(b) \$2500

(c) \$1500

(d) \$2800

(e) \$800

Solution. (d) $\dfrac{3}{4} \times \dfrac{4}{5} \times \dfrac{5}{6} \times (\$5600) = \dfrac{3}{6} \times (\$5600) = \dfrac{1}{2} \times (\$5600) = \$2800.$

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Correct answer = 4, Incorrect answer = −1, Blank = 0

1. A teacher has three packages of stickers. One package contains 56 stickers, another package contains 48 stickers, and the third package contains 58 stickers. If the teacher divides all the stickers equally among 27 students, how many stickers will each student receive?

   (a) 6 stickers
   (b) 9 stickers
   (c) 54 stickers
   (d) 81 stickers
   (e) 162 stickers

   Solution. (a) Adding all stickers together and dividing by 27 we obtain 6.

2. A school district is proposing a 5% increase in the number of days in a school year. Currently there are 180 days in a school year. How long would a school year be with the proposed increase?

   (a) 181 days
   (b) 183 days
   (c) 185 days
   (d) 189 days
   (e) 270 days

   Solution. (d) We first find how much does 5% out of 180 represents: $\dfrac{5}{100} \times 180 = 9$. Then a school year with the proposed increase would have $180 + 9 = 189$ days.

3. Antonio buys 4 notebooks for $3.50 each at a store. The next day he returns to the store and exchanges the notebooks for 3 notebooks that have gone on sale for $1.75 each. Antonio uses the following expression to calculate the amount of money he should receive back from the exchange:

$$(4 \times \$3.50) - (3 \times \$1.75)$$

   Which of the following expressions could Antonio have also used?

(a) $3.50 − $1.75

(b) 3($3.50 − $1.75) + $3.50

(c) (4 × $3.50) − $1.75

(d) (4 × 3) − ($3.50 × $1.75)

(e) 12 − ($3.50 + $1.75)

Solution. (b) $(4 \times \$3.50) - (3 \times \$1.75) = [(1+3) \times \$3.50] - (3 \times \$1.75) = (1 \times \$3.50) + (3 \times \$3.50) - (3 \times \$1.75) = \$3.50 + 3(\$3.50 - \$1.75) = 3(\$3.50 - \$1.75) + \$3.50$.

4. Ann lives in Los Angeles. She would like to use Spring break to visit three of her friends that live in Chicago, New York and San Antonio. How many tours can she possibly make if the order in which she chooses what city to visit next is completely random.

   (a) 3
   (b) 4
   (c) 5
   (d) 6
   (e) 7

   Solution. (d) There are three choices for the first visit and then two choices for the second visit. Thus there are $3 \times 2 = 6$ possible tours that Ann could make.

5. Zach invests $100 in his father's business, and the investment generates a fixed interest of $10 each month. How long does he need to invest in that business in order to raise $1000?

   (a) 80 months
   (b) 90 months
   (c) 100 months
   (d) 110 months
   (e) 120 months

   Solution. (b) 90 months

6. Jack says to his little cousin Ashley: "Right now I am 15 years older than you. In 5 years from now, I will be double your age." How old is Ashley?

   (a) 10
   (b) 15
   (c) 20
   (d) 25
   (e) 30

Solution. **(a)** Let x be Jacks age and let y be Ashleys age. Then we have the following system of equations: $x = y + 15, x + 5 = 2(y + 5)$. Solving for y in the first equation we obtain $y = x - 15$. Plugging $y = x - 15$ into the second equation we obtain $x + 5 = 2(x - 15 + 5)$, which is equivalent to $x + 5 = 2x - 20$. Subtracting x from both sides and adding 20 to both sides we find that $x = 25$. Then $y = 25 - 15 = 10$.

7. An Italian man called Leonardo Fibonacci wants to construct a sequence of natural numbers. He starts with 0 and 1 at positions 0 and 1. To obtain the next number, he adds the two previous ones, so at position 2 he gets $0 + 1 = 1$. What integer will he get at the 15th position?

   (a) 233
   (b) 377
   (c) 610
   (d) 987
   (e) 1597

Solution. **(c)** Let us denote by x_k the number at position k, where $k \geq 0$. Then $x_0 = 0, x_1 = 1$ and $x_2 = 1$. A simple computation shows that

$$x_3 = x_1 + x_2 = 1 + 1 = 2$$
$$x_4 = x_2 + x_3 = 1 + 2 = 3$$
$$x_5 = x_3 + x_4 = 2 + 3 = 5.$$

Continuing in this manner, we find that the term x_{15} at the 15th position is 610.

8. How many three digits car codes can be created if the digits are chosen from $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ and if a repetition of digits is allowed in a code?

   (a) 8^3
   (b) 9^3
   (c) 10^3
   (d) 11^3
   (e) 12^3

Solution. **(c)** There are 10 choices for each digit, thus there are $10 \times 10 \times 10 = 10^3$ three digits car codes that can be created in the manner described above.

9. Pat, Quincy and Rose agree to play a series of games of singles tennis. In each game, the person who won plays the person who sat out the previous game. At the end of the series, Pat has played in 12 games and Quincy has played in 25 games. Who lost game number 7?

   (a) Pat
   (b) Quincy
   (c) Rose

Solution. (c) There were at least 25 games and every player plays in at least every other game. The only way for Pat to play in only 12 games is for Pat to play in games $2, 4, 6, ..., 24$ and lose them all. This means that there were exactly 25 games and Quincy played in all of them, so he must win every game except perhaps the last one. Rose lost game number 7.

10. Line segments $\overline{AB}$ and $\overline{CD}$ each have length 2, are perpendicular, and bisect each other. Each of the four arcs is an arc of a circle, which is tangent to the two line segments. Find the area of the shaded region.

(a) $\dfrac{\pi}{4}$

(b) $4 - \pi$

(c) $1 - \dfrac{\pi}{4}$

(d) $2 + \pi$

(e) π
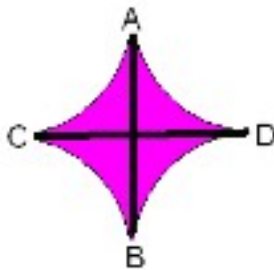
Solution. (b) Let E be the intercept of $\overline{AB}$ and $\overline{CD}$ and F be the center of the circle containing the arc DB. Each of the angles $\angle DEB, \angle EDF$ and $\angle FBE$ is a right angle, and the line segments $\overline{DF}$ and $\overline{FB}$ have the same length, so quadrilateral $FDEB$ is a square with side 1, and so its area is 1. DFB is a quarter-circle, so it has area $\frac{\pi}{4}$. The shaded region DEB has area $1 - \frac{\pi}{4}$, so the entire shaded area is $4 - \pi$.