

CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10
PART I

April 22nd, 2017

1. I'm looking for a number such that its product with 7 added to 3 gives 24. What is my number?

(A) 2

(B) 3

(C) 4

(D) 7

2. A department store uses three times as many five-dollar bills in conducting its daily business as ten-dollar bills. If \$3000 was obtained in five- and ten-dollar bills for the day's business, how many five-dollar bills were obtained?

- (A) 60
- (B) 120
- (C) 180
- (D) 360

3. Suppose that $XYZW$ is a parallelogram whose diagonals intersect at a point A . If $YA = 2t$, $WA = 3t - 4$, and $XZ = 5t$, find XA .

- (A) 20
- (B) 10
- (C) 9
- (D) 4

4. How many ways can we arrange the letters of the word SYSTEM?

- (A) 120
- (B) 360
- (C) 540
- (D) 720

5. The perimeter of a rhombus is 200 feet and one of its diagonals has a length of 80 feet. What is the area of the rhombus?

- (A) 1200 square feet
- (B) 1500 square feet
- (C) 2000 square feet
- (D) 2400 square feet

6. A group of 286 people consists of men, women, and children. There are four times as many men as children and twice as many women as men. How many women are there?

- (A) 176 women
- (B) 88 women
- (C) 44 women
- (D) 22 women

7. Evaluate

$$20 - (3 \times 2^3 - 5).$$

- (A) -191
- (B) 1
- (C) 11
- (D) 131

8. Simplify

$$\left(\frac{-2a^3b^2c}{3a^2b^3c^{-7}} \right)^{-2}.$$

- (A) $\frac{9b^2}{4a^2c^{16}}$
- (B) $\frac{-4a^2c^{16}}{9b^2}$
- (C) $\frac{9a^2}{4b^2c^8}$
- (D) $\frac{4b^2c^8}{9a^2}$

9. Which of the following inequalities has no solution?

(A) $|x + 3| \leq -2$

(B) $|x - 1| > -10$

(C) $\sqrt{7x + 4} > 3$

(D) $\sqrt{-100x + 5} \geq 8$

10. If $f(x) = x - 4$, what is $f(f(3))$?

- (A) 1
- (B) -1
- (C) -5
- (D) 5

11. How many real-valued solutions are there to the equation

$$(x^2 + 1)^2 + 2(x^2 + 1) - 3 = 0?$$

- (A) 0
- (B) 1
- (C) 2
- (D) 4

12. An architect built a model of a 220-foot tall building he is designing. If the model is 25 inches tall and 10 inches wide, how wide is the actual building?

- (A) 44 feet
- (B) 88 feet
- (C) 528 feet
- (D) 1,056 feet

13. David installed a device on his car that is designed to increase his gas mileage by 15%. If he currently gets 22 miles per gallon, what will his gas mileage be after installing the device?

- (A) 3.3 miles per gallon
- (B) 23.3 miles per gallon
- (C) 25 miles per gallon
- (D) 25.3 miles per gallon

14. Esteban is thinking of a number between 20 and 30. The number is prime and not more than 2 away from a perfect square. What is Esteban's number?

- (A) 23
- (B) 27
- (C) 29
- (D) 31

15. If the measures, in degrees, of the three angles of a triangle are x , $x + 10$, and $2x - 6$, the triangle must be

- (A) right.
- (B) equilateral.
- (C) isosceles.
- (D) scalene.

16. What is the slope of a line perpendicular to the line whose equation is $5x + 3y = 8$?

(A) $-\frac{3}{5}$

(B) $\frac{5}{3}$

(C) $\frac{3}{5}$

(D) $-\frac{5}{3}$

17. A sphere is inscribed inside a cube with edges of 6 cm. In cubic centimeters, what is the volume of the sphere?

- (A) $12\pi \text{ cm}^3$
- (B) $36\pi \text{ cm}^3$
- (C) $48\pi \text{ cm}^3$
- (D) $288\pi \text{ cm}^3$

18. Here are the numbers of text messages that students send to each other on a daily basis:

25, 23, 17, 15, 19, 21, 28, 30, 26, 28.

What is the median of these scores?

- (A) 20
- (B) 21
- (C) 23
- (D) 24

19. The number of calories burned while jogging varies directly with the number of minutes spent jogging. If George burns 150 calories by jogging for 20 minutes, how many calories does he burn by jogging for 30 minutes?

- (A) 100
- (B) 180
- (C) 200
- (D) 225

20. Maria has a set of 10 index cards labeled with the digits 0 through 9. She puts them in a bag and selects one at random. Which outcome is most likely to occur?

- (A) Selecting an odd number.
- (B) Selecting a prime number.
- (C) Selecting a number that is at most 5.
- (D) Selecting a number that is divisible by 3.

21. What is

$$\frac{\sqrt{2}}{3\sqrt{2} + \sqrt{8}}?$$

(A) $\frac{1}{5}$

(B) $\frac{1}{3}$

(C) $\frac{\sqrt{5}}{15}$

(D) $\frac{\sqrt{2}}{12}$

22. Julie has three children whose ages are consecutive odd integers. If x represents the age of one of the children, which of the following could NOT be the sum of the ages of the children?

(A) $3x + 6$

(B) $3x$

(C) $3x - 6$

(D) $3x + 1$

23. In $\triangle ABC$, $AB = BC$. An altitude is drawn from B to \overline{AC} , intersecting \overline{AC} at D . Which conclusion is NOT always true?

(A) $AD = BD$

(B) $AD = DC$

(C) $m\angle ABD = m\angle CBD$

(D) $m\angle BDA = m\angle BDC$

24. Solve the equation for x :

$$\frac{x}{x-3} = \frac{3}{x-3} + x - 3.$$

- (A) $x = 0$
- (B) $x = 3$
- (C) $x = 4$
- (D) $x = 3$ or $x = 4$

25. How many whole numbers lie between 4^2 and 4^3 ?

- (A) 49
- (B) 48
- (C) 47
- (D) 46

26. If the numbers $\sqrt[3]{9}$, $\sqrt{5}$, 1, 2, 3 are arranged in order of magnitude, then the middle number is

- (A) $\sqrt[3]{9}$
- (B) $\sqrt{5}$
- (C) 1
- (D) 2

27. The equation

$$(a + b)^2 = a^2 + b^2$$

is

- (A) false for all values of a and b .
- (B) true only if $a = b = 0$.
- (C) true if at least one of a or b is 0.
- (D) true for all values of a and b .

28. Janet made enough applesauce to fill a cylindrical jar with a radius of 6 cm and a height of 12 cm. If she makes the same amount of applesauce and places it in three new jars, each having the same height, but one-third the volume, what is the radius of each new jar?

- (A) $\sqrt{3}$ cm
- (B) $2\sqrt{3}$ cm
- (C) 4 cm
- (D) 6 cm

29. Jose wants to use 40 feet of fencing to enclose a flower garden. Which of these shapes would use all of the fencing and enclose the largest area?

- (A) A rectangle with a length of 8 feet and a width of 12 feet.
- (B) An isosceles right triangle with a side length of about 11.7 feet.
- (C) A circle with diameter of about 12.7 feet.
- (D) A square with side length of 10 feet.

30. How many ways can you choose 12 donuts from 20 different types (assuming that there are at least 12 of each type)?

(A) $\frac{20!}{12!8!}$

(B) $\frac{32!}{12!20!}$

(C) $\frac{31!}{12!20!}$

(D) $\frac{31!}{19!12!}$

Solutions

1 B

2 D

3 B

4 B

5 D

6 A

7 B

8 A

9 A

10 C

11 B

12 B

13 D

14 A

15 D

16 C

- 17 B
- 18 D
- 19 D
- 20 C
- 21 A
- 22 D
- 23 A
- 24 C
- 25 C
- 26 A
- 27 C
- 28 B
- 29 C
- 30 D

CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10
PART II

April 22nd, 2016

1. A half of a third of a fourth of a Coco is equal to a fifth of a sixth of seven Kams. How many Cocos does it take to make eight Kams?

(A) $\frac{1}{630}$

(B) $\frac{5}{28}$

(C) $\frac{32}{35}$

(D) $\frac{10}{7}$

2. A 6-foot man is standing 8 feet horizontally from the base of a light pole. If the man's shadow is 4 feet in length, how far off the ground is the light?

- (A) 24 feet
- (B) 18 feet
- (C) 16 feet
- (D) 12 feet

3. Two men play a card game and must bet one penny for each round. At the end, one man has won three pennies and the other has won three games (but no pennies). How many rounds did they play?

- (A) 3
- (B) 6
- (C) 9
- (D) 12

4. A fenced backyard is in the shape of a regular octagon. The perimeter of the fence is 112 feet. A dog is tied to a corner with a 10-foot chain. How much roaming area inside the fence does the dog have?

- (A) $\frac{75}{2}\pi$ square feet
- (B) $(50\sqrt{2} + 25)\pi$ square feet
- (C) $24\sqrt{2}\pi$ square feet
- (D) 125π square feet

5. Let n be a positive integer. If n is divided by 2, 3, 4, 5, or 6, the remainder is 1, but n is divisible by 7. What is the least possible value of n ?

- (A) 421
- (B) 721
- (C) 301
- (D) 63

6. Suppose a , b , and c are real numbers such that $\frac{a}{b} > 1$ and $\frac{a}{c} < 0$. Which of the following must be true?

- (A) $(a - c)(b - c) > 0$
- (B) $a + b + c > 0$
- (C) $abc > 0$
- (D) $a + b - c > 0$

7. Which of the following CANNOT be the sum of the digits of a square?

- (A) 13
- (B) 11
- (C) 7
- (D) 4

8. How many real roots does the equation

$$\sqrt{x+7} + x = 13$$

have?

- (A) 0
- (B) 1
- (C) 2
- (D) 3

9. If a regular hexagon is inscribed in a circle of radius 3, what is its area?

(A) 3

(B) $\frac{9\sqrt{3}}{2}$

(C) 27

(D) $\frac{27\sqrt{3}}{2}$

10. How many solutions does the equation $||x - 1| - 3| = 3$ have?

- (A) 4
- (B) 3
- (C) 2
- (D) 0

11. Anna read a 480-page book by reading the same number of pages each day. If she had read 8 more pages each day, she would have finished 3 days earlier. How many days did it take Anna to read the book?

- (A) 10
- (B) 12
- (C) 15
- (D) 18

12. The coordinates of three vertices of a parallelogram are $A(1, 1)$, $B(2, 4)$, and $C(-5, 1)$. Compute the area of the parallelogram.

- (A) 9
- (B) 18
- (C) 36
- (D) 72

13. An analog clock displays the time 3:40. What is the measure of the smaller angle formed by the minute and hour hands of the clock?

- (A) 100°
- (B) 110°
- (C) 120°
- (D) 130°

14. What is the smallest positive integer $x > 100$ such that every permutation of the digits of x is prime?

(A) 101

(B) 103

(C) 113

(D) 117

15. A 5-foot tall boy throws a ball 3 feet above his head in the air. The ball bounces to 80% of its previous height with each bounce. What is the total vertical distance the ball travels?

- (A) 48 feet
- (B) 72 feet
- (C) 75 feet
- (D) 80 feet

16. What is the coefficient of x^{25} when $(x^3 + x + 1)^{10}$ is expanded?

- (A) 90
- (B) 130
- (C) 165
- (D) 300

17. Each vertex of a given square is the center of one of four circles. The circles are all congruent and each one is tangent to two others. What is the probability that a randomly chosen point in the figure is inside both a circle and the square?

(A) $\frac{1}{4}$

(B) $\frac{3}{4\pi}$

(C) $\frac{\pi}{3 + 4\pi}$

(D) $\frac{\pi}{4 + 3\pi}$

18. A rectangle has perimeter 10 and diagonal $\sqrt{15}$. What is its area?

- (A) 5
- (B) 15
- (C) 20
- (D) 25

19. Trevor's farm of mutant animals has three-legged goats and five-legged goats. If in one pen, his toddler counts 83 legs and 23 heads, how many 5 legged goats are in the pen?

- (A) 7
- (B) 9
- (C) 16
- (D) 17

20. On a test, the average score for the girls in the class is 91, and the average score for the boys in the class is 85. If the average score for the class is 89, what fraction of the class are boys?

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{2}{5}$

(D) $\frac{3}{5}$

21. If the roots of $x^3 + ax^2 + bx + c$ are three consecutive positive integers, then what are all possible values of $\frac{a^2}{b+1}$?

(A) $\frac{1}{3}$

(B) 1

(C) 3

(D) $1 \leq \frac{a^2}{b+1} \leq 3$

22. Consider a triangular pyramid $ABCD$ with equilateral base ABC of side length 1 such that $AD = BD = CD$ and $m\angle ADB = m\angle BDC = m\angle ADC = 90^\circ$. Find the volume of $ABCD$.

(A) $\frac{2}{24}$

(B) $\frac{\sqrt{3}}{24}$

(C) $\frac{1}{12}$

(D) $\frac{\sqrt{2}}{24}$

23. What is the tens digit of the smallest positive integer that is divisible by each of 20, 16 and 2016?

- (A) 0
- (B) 2
- (C) 4
- (D) 8

24. A total of n points are equally spaced around a circle and are labeled with the integers 1 to n , in order. Two points are called diametrically opposite if the line segment joining them is a diameter of the circle. If the points labeled 7 and 35 are diametrically opposite, then n equals

- (A) 54
- (B) 55
- (C) 56
- (D) 57

25. At Bulldog High School, a total of 36 students are on either the baseball team, the football team, or both. If there are 25 students on the baseball team and 19 students on the football team, how many students play both sports?

- (A) 7
- (B) 8
- (C) 9
- (D) 10

26. A string has been cut into 4 pieces, all of different lengths. The length of each piece is 2 times the length of the next smaller piece. What fraction of the original string is the longest piece?

(A) $\frac{8}{15}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{6}{13}$

27. A merchant wishes to mix candy worth \$5 per pound with candy worth \$2 per pound to get 60 pounds of a mixture that can be sold for \$3 per pound. How many pounds of each type of candy should be used?

- (A) 20 pounds of \$5 candy and 40 pounds of \$2 candy
- (B) 25 pounds of \$5 candy and 35 pounds of \$2 candy
- (C) 30 pounds of \$5 candy and 30 pounds of \$2 candy
- (D) 40 pounds of \$5 candy and 20 pounds of \$2 candy

28. If $x^2yz^3 = 7^4$ and $xy^2 = 7^5$, then $xyz =$

- (A) 7^2
- (B) 7^8
- (C) 7
- (D) 7^3

29. In a convex polygon, exactly five of the interior angles are obtuse. The largest possible number of sides for this polygon is

- (A) 7
- (B) 8
- (C) 9
- (D) 10

30. A store had a sale on t-shirts: for every two t-shirts purchased at the regular price, a third t-shirt would cost \$1. If Tom bought twelve t-shirts for \$120.00, what was the regular price for one t-shirt?

- (A) \$10.00
- (B) \$13.50
- (C) \$14.50
- (D) \$15.00

Solutions

1 D

2 B

3 C

4 A

5 C

6 A

7 B

8 B

9 D

10 B

11 C

12 B

13 D

14 C

15 C

16 A

17 D

18 A

19 A

20 B

21 C

22 D

23 D

24 C

25 B

26 A

27 A

28 D

29 B

30 C