2017
Leap Frog Relay Grades 6-8
Part I Solutions

1. Jaime went to the Big 5 store to buy running shoes. To his pleasant surprise, all running shoes were on a 20% discount sale that day. Jaime also knew that he had to pay a 7.5% sales tax on the discounted price. He had \$60. What is the original (before discount) price of the most expensive shoes he could afford to buy out of the list of prices below?

   a. \$67.42

   b. \$69.50

   c. \$69.76

   d. \$79.78

   e. \$87.50

Correct Answer: **C**. Solution: If x represents the original price of the shoes then $0.8x$ is the discounted price, and 1.075 times $0.8x$ (which is equal to $0.86x$) is the total amount Jaime has to pay. Therefore, $0.86x$ should not be more than \$60, so x should not be more than \$69.76.

2. What is the ratio of the number of even divisors to the number of odd divisors for the number 792?

   a. 3 : 1

   b. 2 : 1

   c. 1 : 1

   d. 1 : 2

   e. 1 : 3

Correct Answer: A. Solution: Since the prime factorization of 792 is (23) (32) (111), 792 has (3+1)(2+1)(1+1) = 24 divisors. The divisors are as follows: 1, 2, 3, 4, 6, 8, 9, 11, 12, 18, 22, 24, 33, 36, 44, 66, 72, 88, 99, 132, 198, 264, 396, 792. The odd divisors are 1, 3, 9, 11, 33, 99, so there are 6 of them. The remaining 18 divisors are even. That

means the ratio of even to odd divisors is 18:6=3:1. (Another way of finding the number of odd divisors is as follows: The prime factorization of 792 is (23) (32) (111). The odd divisors will contain 3 to the power of 0, 1, or 2 (3 choices), and 11 to the power of 0 or 1 (two choices). Therefore, there are 3 times 2 many odd divisors.)

3. How many different teams of 4 representatives can be selected from a group of 7 people?

   a. 24

   b. 28

   c. 35

   d. 840

   e. None of the above

Correct Answer: C. Any of the 7 people can be selected to be the first team member, any of the remaining 6 to be the second, etc. That gives us (7)(6)(5)(4)= 840 choices. However, since it does not matter, who was selected at what time, only that the person is in the team, we should divide this result by (4)(3)(2)(1)= 24 (the different orders of selecting the 4 representatives), so there are only 35 different teams. (Note: "7 choose 4" is 35.)

4. I gave a value to every vertex of a cube. The value of an edge is the sum of the values of the vertices at its ends. The value of a side is the sum of the values of the edges surrounding it. The value of a cube is the sum of the values of its sides. What is the value of the cube if the sum of the values of its vertices is 256?

   a. 768

   b. 1024

   c. 1280

   d. 1536

   e. 2048

Correct Answer: D. Solution: There are 3 edges starting from each vertex of the cube, therefore the sum of the values of the edges is 3 times the sum of the values of the vertices. Each edge is counted as a border of 2 sides, therefore the sum of the values of the sides is 2 times the sum of the values of the edges, so 6 times the sum of the values of the vertices: $6 \times 256 = 1536$.

5. A very thin wire is going upwards in a spiral from the bottom of a cylinder to the top, rising by the same rate all along, making 9 complete rotations around the cylinder. The radius of the cylinder is 12 centimeter, and its height is 2 meter. Which of the following quantities is the closest to the length of the wire?

   a. $3.8m$

   b. $4.5m$

   c. $5.4m$

   d. $6.3m$

   e. $7.1m$

Correct Answer: E. Solution: If we roll out the wire, it will be the hypotenuse of a right triangle where one leg is the height of the cylinder and the other is 9 times the circumference of its circular base. If x is the length of the wire, then $x^2 = (2m)^2 + (9 \cdot 2\pi \cdot 0.12m)^2$. Solving for x, we get x is about $7.1m$.

6. If you roll two fair dice, what is the probability that the product of the two numbers showing up on the top faces is more than 6?

   a. $\frac{7}{12}$

   b. $\frac{11}{12}$

   c. $\frac{7}{18}$

   d. $\frac{11}{18}$

   e. $\frac{11}{36}$

   Correct Answer: D. There are $6 \times 6 = 36$ equally likely outcomes, 22 of which produces a product that is more than 6. Therefore, the probability is $22/36 = 11/18$.

7. The gas consumption of Peter's is car 28 miles per gallon. Juanita's car uses 7 liters of gas on 100 kilometers. Carla's car used 7 gallons of gas on a 300 kilometer trip. (Hint: 1 gallon is 3.6 liter, and 1 mile is 1.6 kilometer.) Which of the following statements is true?

   a. Peter's car uses less gas on 100 miles than Carla's, but more than Juanita's.

   b. Peter's car uses less gas on 100 miles than both Carla's and Juanita's.

   c. Carla's car uses less gas on 100 miles than both Juanita's and Peter's.

   d. Juanita's car uses less gas on 100 miles than Carla's, but more than Peter's.

   e. Juanita's car and Carla's car uses the same amount of gas on 100 miles.

Correct Answer: A. Solution: Peter's car uses about 3.6 gallons on 100 miles, Juanita's about 3.1 gallons, and Carla's about 3.7 gallons.

8. For any positive integer n, define $S(n)$ to be the sum of the positive factors of n. For example, $S(10) = 1 + 2 + 5 + 10 = 18$. Find $S(S(14))$.

   a. 15

   b. 24

   c. 60

   d. 196

   e. 210

Correct Answer: C. $S(14) = 1 + 2 + 7 + 14 = 24$, therefore $S(S(14)) = S(24) = 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60$.

9. The base of an isosceles triangle is 24 inches and its area is 60 square inches. What is the length of one of the congruent sides?

   a. 5 inches

   b. 13 inches

   c. 15 inches

   d. 20 inches

   e. 36 inches

Correct Answer: B. Solution: The area of the triangle is $60 = \frac{24 \cdot height}{2}$. Therefore, the height of the triangle is 5 inches. Because the triangle is isosceles, it can be cut into two congruent right triangles, where one of the legs is the half of the base, and the other leg is the height. Applying the Pythagorean theorem to one of these triangles (122+52=132), we get that the length of the congruent sides (the hypotenuse of the right triangle) is 13 inches.

1		2	
2	3		
			4

10. To complete the grid seen above, each of the digits 1 through 4 must occur once in each row and once in each column. What number will occupy the lower right-hand cell?

    a. 1

    b. 2

    c. 3

    d. 4

    e. Cannot be determined

Correct Answer: B. The number in the last column of the second row must be 1 because there are already a 2 and a 3 in the second row and a 4 in the last column. By similar reasoning, the number above the 1 must be 3. So the number in the lower right-hand cell must be 2. This is not the only way to find the solution.

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Part II Solutions

No calculators allowed
Correct Answer = 4 points
Incorrect Answer = −1 point
Blank = 0 points

11. Esperanza is dropping a ball and watching it bounce. If she drops the ball from the top of an 18 foot playground, and every time it rebounds to half the height it was at before, how far does the ball travel by the time it hits the ground for the third time? (The ball bounces straight up and straight down.)

    a. 54 feet

    b. 9 feet

    c. 31.5 feet

    d. 27 feet

    e. 45 feet

Correct Answer: e. Solution: The ball travels 18 feet down, hits the ground, bounces up 9 feet, falls down 9 feet, hits the ground a second time, then bounces up 4.5 feet, falls down 4.5 feet, and hits the ground a third time. In total: $18 + 9 + 9 + 4.5 + 4.5$ feet $= 45$ feet.

12. Sabrina runs at 7 miles per hour for 1 minute, then walks at 3 miles per hour for 1 minute, then runs at 5 miles per hour for 2 minutes, then walks at 3 miles per hour for 1 minute. She repeats this 5 times. What was the total distance she ran and walked?

    a. 15 miles

    b. 15/6 miles

    c. 23/12 miles

    d. 18 miles

    e. 2 miles

Correct Answer: c. Solution: She runs the following distances:

- 7 miles per hour for 5 minutes: 7/12 miles
- 3 miles per hour for 10 minutes: 3/6 miles
- 5 miles per hour for 10 minutes: 5/6 miles
- Total: $7/12 + 6/12 + 10/12 = 23/12$ miles or $1\frac{11}{12}$miles

13. Suppose Addi, Manit, and Gaby are making lemonade. They want to choose from the following recipes to make the recipe that tastes the most strongly of lemon. Which one should they choose?

    a. 3 cups of water to 2 scoops of lemonade mix

    b. 1 cup of water to $\frac{3}{4}$ scoops of lemonade mix

    c. 10 cups of water to 7 scoops of lemonade mix

    d. 6 cups of water to 5 scoops of lemonade mix

    e. 13 cups of water to 9 scoops of lemonade mix

Correct Answer: d. Solution: There are several ways to think of this one, but if you take the fraction of lemonade scoops to water, 5/6 is the largest fraction.

14. Suppose we have a square with side length S, and a circle with diameter S. If we take the area of the circle divided by the area of the square, what should the result be?

    a. π

    b. There is not enough information to get a result.

    c. $1/\pi$

    d. $\pi/4$

    e. $1/3$

Correct Answer: d. Solution: The area of the circle will be $A_{circle} = \pi\left(\frac{S}{2}\right)^2 = \frac{\pi S^2}{4}$, and the area of the square will be $A_{square} = S^2$. So the division will be $\frac{\frac{\pi S^2}{4}}{S^2} = \frac{\pi S^2}{4S^2} = \pi/4$.

15. A perfect number is one where if you add up the factors of the number, except the number itself, then the sum is the number again. For example the factors of 6 are 1, 2, and 3, and $1 + 2 + 3 = 6$. Which of the numbers below is perfect?

    a. 28

    b. 29

    c. 30

    d. 31

    e. 32

Correct Answer: a. Solution: Because the factors of 28 are 1, 2, 4, 7, 14, and $1 + 2 + 4 + 7 + 14 = 28$.

16. A particular kind of plant triples in height over the course of one week. Shaina buys one of these plants. If at the end of the third week that she had the plant, Shaina measured that it was 54 cm tall, then how tall was the plant when she first bought it?

    a. 1 cm

    b. 2 cm

    c. 4 cm

    d. 6 cm

    e. 8 cm

Correct Answer: b. Solution: It was 2 cm because then at the end of 1 week, it would be 6 cm. At the end of 2 weeks it would be 18 cm. And at the end of the third week, it would be 54 cm.

17. Six college students volunteered to clean up trash in a particular abandoned lot. They told the coordinator that they could clean it up in 4 days. The coordinator said he wanted the lot cleaned up in three days. How many more students would have to join to accomplish this?

    a. 1

    b. 2

    c. 3

    d. 4

    e. 5

Correct Answer: b. Solution: 2 more students. Six students multiplied by 4 days means it takes 24 student-days to clean up the lot. Then if there were eight students multiplied by 3 days, it would be an equivalent amount of work.

18. Lillian has a ribbon that is 2 and $\frac{3}{4}$ feet long. She wants to cut pieces that are $\frac{1}{2}$ of a foot long to give to her friends. How many pieces can she cut?

a. 2 pieces

b. 4 $\frac{3}{4}$ pieces

c. 4 $\frac{1}{2}$ pieces

d. 5 $\frac{1}{4}$ pieces

e. 5 $\frac{1}{2}$ pieces

Correct Answer: e. Solution: There are 4 half foot pieces inside of the 2 foot part, an extra half foot piece in the $\frac{3}{4}$ and half of a half foot (1/4 of a foot) piece left.

19. How many steps are required to break a 6×8 sized bar of chocolate into 1×1 pieces? You can break an existing piece of chocolate horizontally or vertically. You cannot break two or more pieces at once (so no cutting through stacks).

a. 14

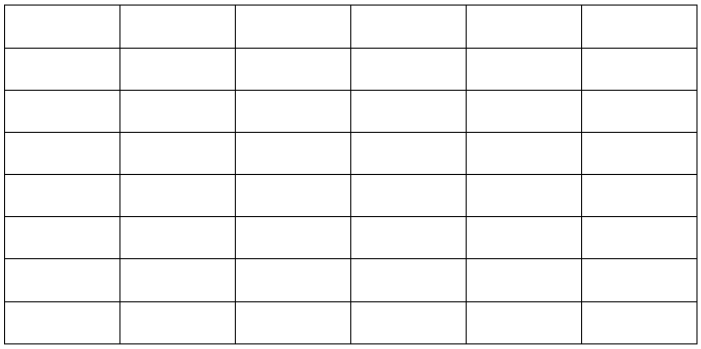
b. 16

c. 47

d. 48

e. 49

Correct Answer: c. Solution: If you imagine breaking the table below vertically, that would be 5 breaks to make 6 columns of 8 squares. Then one of the columns would take 7 breaks to break into individual pieces; thus, altogether there are $6 \times 7 + 5 = 42 + 5 = 47$.



20. Suppose that Daya has a $4 \times 4 \times 4$ inch solid wood cube. Daya puts the cube on a worktable and paints all of the sides except the bottom purple. A day later, she cuts the cube into $1 \times 1 \times 1$ inch cubes. How many of the small cubes have exactly two sides painted?

    a. 12 small cubes

    b. 16 small cubes

    c. 18 small cubes

    d. 20 small cubes

    e. 24 small cubes

Correct Answer: d. Solution: You would get the edges of the top layer minus the corners to get 8 cubes. Then for the rest, each vertical edge, again minus the top corner, would produce three cubes with two sides painted. In total $8 + 12 = 20$.