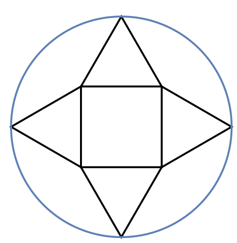
2017
Leap Frog Relay Grades 11-12
Part I

No calculators allowed
Correct Answer = 4, Incorrect Answer = −1, Blank = 0

1. If r_1 and r_2 are the two real number solutions to the equation $x^2 + x = 2017$, then $(r_1 + r_2)^{2017} =$ ________.

   (a) 0
   (b) 1
   (c) 2^{2017}
   (d) -2^{2017}
   (e) None of these

2. The central square is sharing its sides with 4 equilateral triangles, and the combined figure is inscribed in the circle as pictured below.



   What is the ratio of circle area to square area?

   (a) $\sqrt{6}\pi$
   (b) $\pi\left(1 + \frac{\sqrt{3}}{2}\right)$
   (c) 2π
   (d) $\pi\left(1 + \sqrt{3}\right)$
   (e) None of these

3. If you triple the radius of a circle, then the resulting *percentage* increase in circle area is ________.

(a) 300%

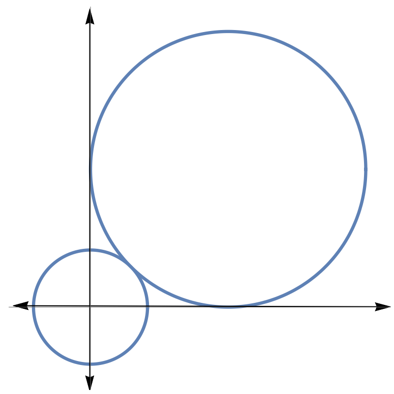
(b) 600%

(c) 800%

(d) 900%

(e) None of these

4. In the figure below, the smaller circle is centered at the origin and has radius equal to a, while the larger circle is mutually tangent to the smaller circle and the two coordinate axes, with radius equal to b. Then, $b/a =$ ________.



(a) $\dfrac{3}{2}$

(b) 2

(c) $1 + \sqrt{2}$

(d) $\dfrac{5}{2}$

(e) None of these

5. If $\log_{4034} 2 = a$, then $\log_{2017} 4034 =$ ________.

(a) $\dfrac{1}{a}$

(b) $\dfrac{1}{1+a}$

(c) $\dfrac{1}{2a}$

(d) $\dfrac{2}{1+a}$

(e) None of these

6. If $\sqrt[3]{4} \cdot \sqrt[4]{x} = 2\sqrt[12]{32}$, then $x =$ ________.

(a) 64 (b) 8

(c) 4 (d) 32

(e) None of these

7. If $\sin(x + \pi) = \sin(x + \pi/2)$ and $0 < x < \pi$ (x is measured in radians), then $x =$ ________.

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

(e) None of these

8. Suppose N is the smallest integer larger than 1 such that when divided by *every* $k = 2, 3, \ldots, 10$, the resulting remainder is 1. Then, $\ldots$.

(a) $500 < N < 1000$ (b) $1000 < N < 1500$

(c) $1500 < N < 2000$ (d) $2000 < N < 2500$

(e) None of these

9. Define a function f on *positive integers* by

$$f(x) = \begin{cases} x/2 & \text{if } x \text{ is even,} \\ 3x + 1 & \text{if } x \text{ is odd.} \end{cases}$$

How many (integer) solutions are there to the equation

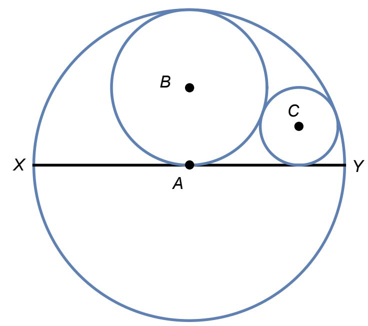
$$f(x) + f(x + 1) = 2017?$$

(a) 0 (b) 1

(c) 2 (d) 3

(e) None of these

10. Let's label the three circles pictured below by their respective centers A, B, and C. Circle B is tangent to circle A and goes through the center point A and is tangent to the diameter $\overline{XY}$ of circle A. Circle C is mutually tangent to circles A and B and the diameter $\overline{XY}$. If the radius of circle A is R, then the radius of circle C is ______.



(a) $\dfrac{R}{2\sqrt{2}}$

(b) $\dfrac{R}{4}$

(c) $\dfrac{R}{2+\sqrt{2}}$

(d) $\dfrac{R}{4\sqrt{2}}$

(e) None of these

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11. The positive real number solution to the equation

$$\frac{x}{2017} - \frac{2017}{x} = 1$$

is . . .

(a) $x = 2017(\sqrt{5} + 1)$

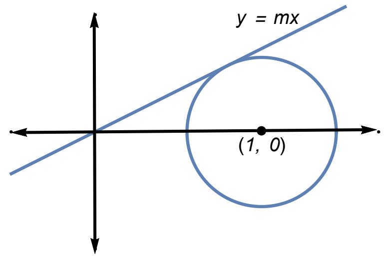
(b) $x = 2017(\sqrt{5} - 1)$

(c) $x = \dfrac{\sqrt{5} - 1}{2017}$

(d) $x = \dfrac{\sqrt{5} + 1}{2017}$

(e) None of these

12. In the figure below, the circle centered at the point $(1, 0)$ is tangent to the line $y = mx$, where $m > 0$. Then, the radius of the circle is ________.

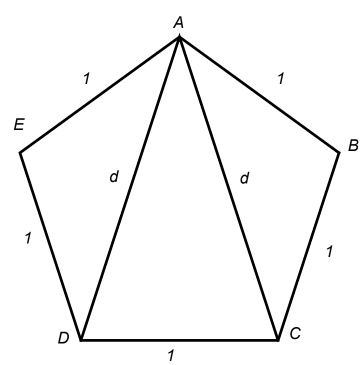


(a) $\dfrac{1}{\sqrt{m^2+1}}$ (b) $\dfrac{m+1}{\sqrt{m^2+1}}$

(c) $\dfrac{m^2}{\sqrt{m^2+1}}$ (d) $\dfrac{m}{\sqrt{m^2+1}}$

(e) None of these

13. The pentagon $ABCDE$ pictured below is a *regular* pentagon with all five side lengths equal to 1. Let $d = AC = AD$. Then, $d =$ ________.



(a) $\dfrac{\sin 108^\circ}{\sin 36^\circ}$ (b) $\dfrac{2\sin 108^\circ}{\sin 36^\circ}$

(c) $\dfrac{\sin 108^\circ}{2\sin 36^\circ}$ (d) $\dfrac{2\sin 108^\circ}{3\sin 36^\circ}$

(e) None of these

14. How many multiples of 2017 with the units digit equal to 1 are there between 0 and 20,172,017?

(a) 999

(b) 1000

(c) 1001

(d) 1002

(e) None of these

15. The solution to the inequality $-1 \leq |x-2| - |x-4| \leq 1$ is in the form $a \leq x \leq b$. Then, $a + b =$ ________.

(a) 4

(b) 5

(c) 6

(d) 7

(e) None of these

16. A one percent increase in the diagonal length of a square results in what percentage increase in its area?

(a) 1.99%

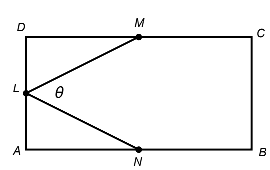
(b) 2%

(c) 2.01%

(d) 2.02%

(e) None of these

17. In the rectangle $ABCD$ pictured below, $AB = DC = a$, $AD = BC = b$, and L, M, N are the respective midpoints of $\overline{AD}, \overline{DC}, \overline{AB}$. Let $\theta = m\angle MLN$. Then, $\cos\theta =$ ________.



(a) $\dfrac{a-b}{a+b}$

(b) $\dfrac{a}{b}$

(c) $\dfrac{a^2-b^2}{a^2+b^2}$

(d) $\dfrac{a^2+b^2}{a^2-b^2}$

(e) None of these

18. Lenny has \$5.85 in nickels, dimes and quarters in his pocket. Assuming he has 52 coins, what is the least number of nickels he could have?

(a) 1 (b) 2

(c) 3 (d) 4

(e) None of these

19. If you divide 2017 by 20, there results the remainder 17. Find the number of integers m larger than 17 (and smaller than 2017) for which if you divide 2017 by m, there results the remainder 17.

(a) 11 (b) 12

(c) 13 (d) 14

(e) None of these

20. Suppose a, b, c, d are positive real numbers. Then,

$$\log_{\left(a^b\right)}\left(c^d\right) = \underline{\qquad\qquad}.$$

(a) $\dfrac{d \log_a c}{b}$ (b) $\dfrac{d \log_a c}{\log_a b}$

(c) $\dfrac{d \log_a c}{\log_b a}$ (d) $\dfrac{d \log_b c}{\log_a b}$

(e) None of these