2012
Leap Frog Relay Grades 6-8
Part I Solutions

No calculators allowed
Correct answer = 4, Incorrect answer = −1, Blank = 0

1. The top of one tree is 8 feet higher than the top of another tree. The heights of the two trees are in the ratio $7 : 9$. In feet, how tall is the shorter tree?
   (a) 15 feet
   (b) 24 feet
   (c) 28 feet
   (d) 56 feet
   (e) 63 feet

   Solution. (C) The sum of the heights of the two trees can be divided into 16 parts where two parts is 8 feet, so one part is 4 feet. The shorter tree has 7 parts so its height is 7×4 feet $= 28$ feet.

2. What is the product of the positive factors of 99?
   (a) 99
   (b) 99^3
   (c) 99^6
   (d) 9^{11}
   (e) 9^{33}

   Solution. (B) Since $99 = 3 \times 3 \times 11$, the positive factors of 99 are $1, 3, 9, 11, 33, 99$. The product of these factors is 99^3.

3. A straight line on a graph passes through the points $(4, 5)$ and $(6, 2)$. Which of these points also lies on the line?
   (a) $(10, 7)$
   (b) $(2, −3)$
   (c) $(8.5, −2)$
   (d) $(9, −2.5)$
   (e) $(−4, 5)$

   Solution. (D) The slope of the line is $\dfrac{(2 − 5)}{(6 − 4)} = −1.5$. Therefore, for every one unit increase in the first coordinate will result in 1.5 units decrease in the second coordinate.

4. The average cost of a long-distance call in the USA in 1995 was 21 cents per minute, and the average cost of a long-distance call in the USA now is 6 cents per minute. Find the approximate percent decrease in the cost per minute of a long-distance call.
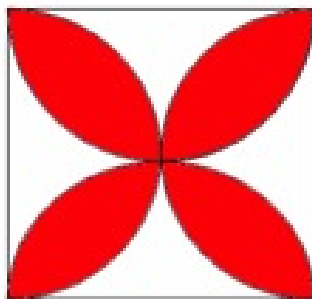   (a) 15%
   (b) 27%
   (c) 71%

(d) 126%
(e) 350%

Solution. (C) The difference in the cost of long distance call per minute from 1995 to now is $21 - 6 = 15$ cents. The percent decrease is $100 \times 15/21$, which is about 71%.

5. A rubber ball rebounds to half the height it drops. If the ball is dropped from a rooftop 18 m above the ground, what is the total distance traveled by the time it hits the ground the third time?
   (a) 31.5 m
   (b) 40.5 m
   (c) 45 m
   (d) 54 m
   (e) 63 m

Solution. (C) The ball falls down 18 meters and hits the ground first. Then it bounces back to height 9 m, falls down 9 m, and hits the ground the second time. Then it bounces back to height 4.5 m, and falls down 4.5 m to hit the ground the third time. The total distance traveled is $18 + 9 + 9 + 4.5 + 4.5 = 45$ m.

6. Four semi-circles are drawn with centers at the midpoint of the sides of a 2 inches by 2 inches square. What is the area of the shaded region in square inches?
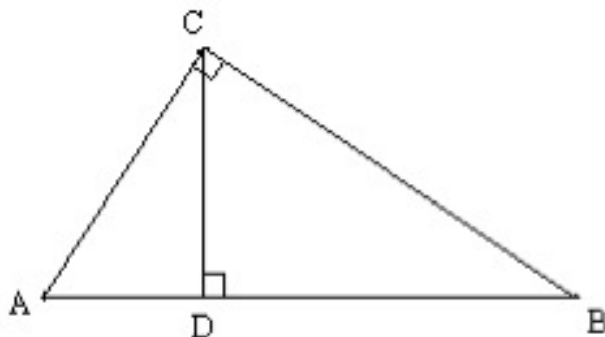

   (a) $4 + 2\pi$
   (b) $4\pi + 2$
   (c) $4\pi - 4$
   (d) $2\pi - 4$
   (e) $8\pi + 4$

Solution. (D) The shaded region looks like a flower with 4 equally big petals. The area of one petal is the difference between the area of a semi-circle with radius 1 inch and the area of the isosceles right triangle inscribed into the semicircle. The leg of the right triangle is the half of the diagonal of the square, so it is the square root of 2 inches long. Therefore, the area of the shaded region is $4(\frac{\pi}{2} - 1) = 2\pi - 4$ square inches.

7. What is the sum of all distinct four-digit numbers that contain only the digits $1, 2, 3, 4,$ or 5, each at most once? (Hint: The numbers 3521 and 3215 are distinct four-digit numbers that should be considered in the sum, but 3321 or 3721 are not good four-digit numbers for our sum, since the first has more than one 3 in it, and the second has a 7 in it.)

   (a) 225000
   (b) 399960
   (c) 450000
   (d) 543210
   (e) 720360

Solution. **(B)** There are $5! = 120$ "good" four-digit numbers we can form. The average value of these numbers is 3333. Therefore, the sum is $120 \times 3333 = 399960$.

8. If $AC = 8$ cm and $AB = 10$ cm, find the measure of the segment CD in the figure below (the figure is not drawn to scale).

   (a) 4.8 cm
   (b) 5.4 cm
   (c) 6.0 cm
   (d) 6.8 cm
   (e) 7.2 cm

Solution. **(A)** Since ABC is a right triangle, we can use the Pythagorean theorem to get $BC = 6$ cm. Since triangles ABC and ACD are similar, $CB : AB = CD : AC$. Therefore, $CD = CB \times AC/AB = 6 \times 8/10 = 4.8$ cm.

9. One hundred students at Century Middle School participated in the local math competition last year, and their mean score was 100. The number of non-8th graders participating in the competition was 50% more than the number of 8th graders, and the mean score of the 8th graders was 50% higher than that of the non-8th graders. What was the mean score of the 8th graders?

   (a) 120
   (b) 125
   (c) 133
   (d) 150
   (e) 175

Solution. (B) If n represents the number of 8th graders, then $n + 1.5n = 100$ yields $n = 40$. So there are 40 8th graders and 60 non-8th graders. If s represents the average score of non-8th graders, then the mean score of 8th graders is $1.5s$ and the equation for the total scores is $60s + 40(1.5s) = 10000$. From here, $1.5s = 125$.

10. You can get 2 sandwiches for the price of 5 sodas. A person bought 3 sandwiches and 3 sodas for $15.75. What is the difference between the price of a sandwich and the price of a soda?
    (a) $1.25
    (b) $1.50
    (c) $1.75
    (d) $2.00
    (e) $2.25

Solution. (E) The price of a soda is two fifth of the price of a sandwich. If p represents the price of a sandwich in dollars, then $3p + \frac{2}{5}(3p) = 15.75$. Solving this equation for p, we get $p = \$3.75$. The price of a soda is then $1.50. Therefore, the difference between the prices is $2.25.

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Part II Solutions

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Correct answer = 4, Incorrect answer = −1, Blank = 0

1. A purse that was originally $300, was reduced in price by 10% each month for three months in a row. What is the new price of the purse after the three months?
   (a) $210
   (b) $218.70
   (c) $243
   (d) $243.75
   (e) $270

   Solution. (B) $0.9 \cdot (\$300) = \$270$; $0.9 \cdot (\$270) = \$243$; $0.9 \cdot (\$243) = \$218.70$.

2. A company wants to make a coffee blend of Sumatra, Columbian and Kenyan beans, that will have a final cost of $2.50 a pound. How many pounds of Sumatra beans that cost $3 a pound must be added to 20 pounds of Columbian beans that cost $2 a pound and 5 pounds of Kenyan beans that cost $4 a pound?
   (a) 7.5
   (b) 3
   (c) 28
   (d) 5
   (e) 13

   Solution. (D) 20 pounds of Columbian beans and 5 pounds of Kenyan beans totals 25 pounds, and costs $60. Denote by x the number of pounds of Sumatra beans that must be added. Then $3x + 2 \cdot 20 + 4 \cdot 5 = (2.5) \cdot (25 + x)$.

   $$\frac{60 + 3x}{25 + x} = 2.5$$

   $$62.5 + 2.5x = 60 + 3x$$

   $$2.5 = 0.5x$$

   $x = 5$ lbs. of Sumatra beans are needed.

3. Find a number w such that the three points $(-5, w), (-3, 4)$ and $(-1, -2)$ all lie on the same line.
   (a) 15
   (b) 10
   (c) −5
   (d) −10
   (e) 12

Solution. (B) Using the two points to find the equation of the line gives: $y = -3x - 5$. Plugging in -5 for x gives the result: $y = 15 - 5 = 10$.

4. One-square-foot floor tiles come 25 to a package. Find out how many packages need to be purchased in order to cover a rectangular floor 27 feet by 20 feet.
   (a) 540
   (b) 21
   (c) 22
   (d) 23
   (e) 515

   Solution. (C) $27 \times 20 = 540 \text{ ft}^2$. Then $\dfrac{540}{25} = 21\dfrac{15}{25}$ packages, and therefore we need to buy 22 packages.

5. Twice the difference of a number and 5 is the same as 2 added to four times the number. Find the number.
   (a) -3
   (b) 2
   (c) -4
   (d) 5
   (e) -6

   Solution. (E) Denote the number by x. Then we have:
   $$2(x - 5) = 2 + 4x$$

   $$2x - 10 = 2 + 4x$$

   $$-2x = 12$$

   $$x = -6$$

6. A group of dogs and human pet owners have 148 legs and 60 heads. How many dogs and humans are in their respective groups?
   (a) 46 dogs and 14 humans
   (b) 13 dogs and 47 humans
   (c) 14 dogs and 46 humans
   (d) 43 dogs and 17 humans
   (e) 37 dogs and 30 humans

   Solution. (C) Let $d = \#$ of dogs and $h = \#$ of humans; then since dogs have 4 legs and humans have two legs, and dogs and humans each have one head, then:
   $$4d + 2h = 148$$
   $$d + h = 60$$

   Letting $h = 60 - d$ and substituting, gives
   $$4(60 - h) + 2h = 148$$
   $$240 - 4h + 2h = 148$$
   $$240 - 2h = 148$$
   $$92 = 2h$$

$h = 46$ humans. So, $d = 60 - 46 = 14$ dogs.

7. Which number is a multiple of 9?
   (a) 111111112
   (b) 131316453
   (c) 161616161
   (d) 191919191
   (e) 232828888

   Solution. (**B**) Number fact: numbers are divisible by 9 if and only if their digits add to a multiple of 9.

8. Suppose your scores on 9 out of 10 quizzes in a math class were:

Quiz #	1	2	3	4	5	6	7	8	9	10
Score out of 15 pts.	10	8	12	6	13	9	11	12	13	x

   What is the minimum score you need on the 10th quiz in order to guarantee you pass the class with at least a 70% average?
   (a) 11
   (b) 12
   (c) 13
   (d) 14
   (e) 15

   Solution. (**A**) $\dfrac{10 + 8 + 12 + 6 + 13 + 9 + 11 + 12 + 13 + x}{10 \cdot 15} = \dfrac{94 + x}{150} = 0.70 \implies$
   $94 + x = 105 \implies x = 11.$

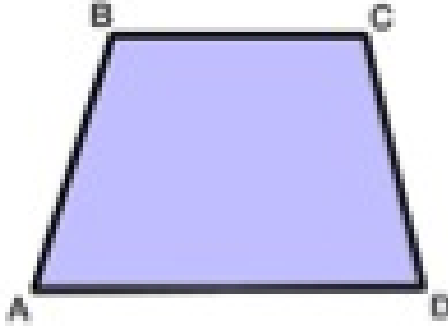9. In the following problem, Z represents a missing exponent. Determine the value of Z:

   $$\frac{9j^Z}{(-3j^2x^2)^5} = \frac{-1}{27j^7x^{10}}$$

   (a) 0
   (b) 2
   (c) 3
   (d) -1
   (e) -2

   Solution. (**C**) Applying the rules for exponents results in the need for $j^{Z-10} = j^{-7} \implies Z = 10 + (-7) = 3.$

10. Find the area of the following isosceles trapezoid given that the slanted sides have length $|\overline{AB}| = |\overline{DC}| = 10$ and $|\overline{BC}| = 5$ and $|\overline{AD}| = 17$.
    (a) 20
    (b) 176
    (c) 88
    (d) 68
    (e) 48

Solution. **(C)** $17 - 5 = 12 \implies$ base of two right triangles is 6, so using the Pythagorean theorem, the height of the trapezoid is $h^2 = 10^2 - 6^2 = 64 \implies h = 8$.
$A = \frac{1}{2}(5 + 17) \cdot 8 = 88$.