CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10
PART I

April 16th, 2011

1. What is
$$5 + 4 \div 2 - 3 \times 2?$$

   (a) 1
   (b) 3
   (c) 8
   (d) $-\dfrac{3}{2}$

2. Mary has part of a roll of quarters for the arcade. She gives half of them to her sister, Sue. After she gives another 2 quarters to Martha, she has 6 quarters left. How many quarters did Mary start with?

   (a) 20
   (b) 16
   (c) 14
   (d) 12

3. A circular swimming pool is 6 feet in diameter. If the tile surrounding the pool is 3 inches wide, what is the circumference of the circle made by the tile?

   (a) 18π feet
   (b) 12π feet
   (c) $\dfrac{25}{2}\pi$ feet
   (d) $\dfrac{13}{2}\pi$ feet

4. Solve the following system of equations

$$4r + 3t = 4$$
$$4r = 3t - 8$$

(a) $r = 2,\ t = -\dfrac{4}{3}$

(b) $r = 10,\ t = 16$

(c) $r = -\dfrac{1}{2},\ t = 2$

(d) no solution

5. How many integers x satisfy

$$|x| < \pi?$$

(a) 3

(b) 4

(c) 7

(d) More than 7

6. If $2 < x < 4$ and $3 < y < 7$, what is the largest integer value of $x + y$?

(a) 10

(b) 9

(c) 8

(d) 7

7. If $0 < a < b < 1$, which of the following is (are) true?

I. $a - b < 0$

II. $\dfrac{1}{ab} > 1$

III. $\dfrac{1}{b} - \dfrac{1}{a} > 0$

(a) I only

(b) II only

(c) I and II only

(d) I, II, and III

8. Simplify

$$\left(-\frac{3x^{-2}}{4y^{-3}z^2}\right)^4.$$

(a) $-\dfrac{81x^2}{256yz^8}$

(b) $\dfrac{81x^2}{256yz^8}$

(c) $-\dfrac{81y^{12}}{256x^8z^8}$

(d) $\dfrac{81y^{12}}{256x^8z^8}$

9. If $7^a7^b = \dfrac{7^c}{7^d}$, what is d in terms of a, b, and c?

(a) $\dfrac{c}{ab}$

(b) $c - a - b$

(c) $a + b - c$

(d) $c - ab$

10. If x and y are positive integers, and $(13^x)^y = 13^{13}$, what is the average of x and y?

(a) $\dfrac{13}{2}$

(b) 13

(c) 7

(d) 14

11. What is the coefficient of x^2y^4 after expanding $(x - 2y)^6$?

(a) 240

(b) 60

(c) -12

(d) -160

12. For the final step in a calculation, Paul accidentally divided by 1000, instead of multiplying by 1000. What should he do to correct his error?

 (a) Multiply by 1000.
 (b) Multiply by 10,000.
 (c) Multiply by 100,000.
 (d) Multiply by 1,000,000.

13. If p and q are prime numbers larger than 2, which of the following **must** be true?
    I. $p + q$ is even.
   II. pq is odd.
  III. $p^2 - q^2$ is even.

 (a) I only.
 (b) II only.
 (c) I and II only.
 (d) I, II, and III.

14. Billy won some goldfish at the state fair. During the first week, $\frac{1}{5}$ of the goldfish died; and during the second week, $\frac{3}{8}$ of the remaining goldfish died. What fraction of the original goldfish were still alive at the end of two weeks?

 (a) $\frac{3}{10}$

 (b) $\frac{37}{40}$

 (c) $\frac{1}{2}$

 (d) $\frac{1}{8}$

15. Given square $ABCD$, where $AB = 2$, and equilateral triangle AED inside $ABCD$, determine the area between $ABCD$ and AED.

 (a) $\sqrt{3}$
 (b) 3
 (c) $4 - \sqrt{3}$
 (d) $4 + \sqrt{3}$

16. Marta's average on her six mathematics exams this semester is 75. Fortunately for Marta, her teacher drops each student's lowest grade, thus raising Marta's average to 85. What was her lowest grade?

(a) 20
(b) 25
(c) 30
(d) 40

17. What is the minimum number of rectangular tiles, each 12 cm by 18 cm, needed to completely cover 5 flat rectangular surfaces, each 60 cm by 180 cm?

(a) 50
(b) 100
(c) 200
(d) 250

18. If $r < 0$ and $(4r - r)^2 = 36$, what is the value of r?

(a) -2
(b) -1
(c) $-\dfrac{1}{2}$
(d) $-\dfrac{1}{4}$

19. Pedro has excactly 4 blue pens, 8 black pens, and 3 red pens in his backpack. If he pulls out one pen then, keeping that one out for himself, pulls a second pen out (both without looking), what is the probability that both pens are black?

(a) $\dfrac{4}{15}$

(b) $\dfrac{64}{225}$

(c) $\dfrac{16}{105}$

(d) $\dfrac{32}{225}$

20. Two hot dogs and a soda cost $3.25. If 3 hot dogs and a soda cost $4.50, how much do 2 sodas cost?

(a) $0.75
(b) $1.25
(c) $1.50
(d) $2.50

21. A piece of wire is formed in the shape of a semicircle of diameter 12. If the piece of wire is then bent to form a circle, without any of the wire overlapping, what is the area of the circle?

(a) 6π
(b) 9π
(c) 12π
(d) 18π

22. A store determines the retail price of a DVD player by marking up its wholesale price by 60%. After a 25% discount off of the DVD player's retail price, the DVD player costs $162. What is the wholesale price of the DVD player?

(a) $54
(b) $81
(c) $135
(d) $216

23. If a line with a slope of two passes through points $(2, 6)$ and $(-2, -y)$, what is the value of y^2?

(a) 0
(b) 2
(c) 4
(d) 8

24. Max decides to carve out a cylinder vertically in the center of a wooden cube. The cube has a side length of 6 cm, and the cylinder has a radius of 2 cm. What is the surface area of the cube after the cylinder has been carved out?

(a) $216 - 4\pi$
(b) $216 + 16\pi$
(c) $144 + 24\pi$
(d) $216 - 16\pi$

25. If a, b, and c are positive consecutive even integers, and $c > b > a$, what is the value of $b^2 - ac$?

(a) -4
(b) $a^2 - 4$
(c) 4
(d) $a^2 + 4$

26. What is the value of the sum of the external angles of a triangle subtracted from the sum of the external angles of a pentagon?

(a) 0
(b) 30
(c) 90
(d) 180

27. If a is 30% greater than A and b is 20% greater than B, then ab is what percent greater than AB?

(a) 25%
(b) 50%
(c) 56%
(d) 60%

28. If John's age is increased by Mary's age, the result is 2 times John's age 3 years ago. If Mary is now 16 years old, what is John's present age?

    (a) 19
    (b) 22
    (c) 26
    (d) 33

29. Two boys can paint a fence in 5 hours. How long would it take 3 boys to paint 2 fences?

    (a) 3 hours
    (b) 3 hours 20 minutes
    (c) 6 hours
    (d) 6 hours 40 minutes

30. If ℓ_1 is a horizontal line passing through (1, 8) and ℓ_2 is a vertical line passing through (-3, 4), at what point do ℓ_1 and ℓ_2 intersect?

    (a) (-3, 8)
    (b) (1, 4)
    (c) (-1, 6)
    (d) (0, 0)

31. What is the units digit of 3^{36}?

    (a) 1
    (b) 3
    (c) 7
    (d) 9

32. If $f(x) = x + 2$ and $f(g(1)) = 6$, which of the following could be $g(x)$?

(a) $3x$
(b) $x + 3$
(c) $x - 3$
(d) $2x + 1$

33. When n is divided by 12, the remainder is 6. What is the remainder when n is divided by 6?

(a) 0
(b) 1
(c) 2
(d) 3

34. A certain class has 6 girls and 5 boys. Four of these students are to line up in the front of the room, with one girl on each end and two boys in between. How many such arrangements are possible?

(a) 20
(b) 200
(c) 462
(d) 600

35. Trent walked from John's house 3 miles due north then 4 miles due east, while John walked 6 miles due south and 8 miles due west. If John were to walk directly to Trent's location, how many hours would it take him if John walks at a rate of 3 miles per hour?

(a) 4
(b) 5
(c) 10
(d) 14

36. If $\ln 6 = 1.7918$ and $\ln 2 = 0.6931$, then $\ln 3 =$?

    (a) 1.0987
    (b) 1.2419
    (c) 2.4849
    (d) 2.5852

37. The second term of an arithmetic sequence is -1 and the eighteenth term is 47. Find the seventh term of the sequence.

    (a) -4
    (b) 4
    (c) 7
    (d) 14

38. A rubber ball is droppped from a height of 30 feet to the pavement, and the rebound is one-fourth the distance it drops. If after each descent the ball continues to rebound to one-fourth the distance it drops, what is the total distance the ball travels?

    (a) 50 feet
    (b) 40 feet
    (c) 35 feet
    (d) 34 feet

39. The base of triangle T is 40% less than the length of rectangle R. The height of T is 50% greater than the width of R. The area of triangle T is what percent of the area of rectangle R?

    (a) 10%
    (b) 45%
    (c) 90%
    (d) 110%

40. If $\dfrac{4\sqrt{x+2}}{y-3} = 8y$, what is x in terms of y?

(a) $2y^2 - 12y - 2$
(b) $4y^4 - 36y^2 - 2$
(c) $4y^4 - 12y^3 - 36y^2$
(d) $4y^4 - 24y^3 + 36y^2 - 2$

Solutions

1. A
2. B
3. D
4. C
5. C
6. A
7. C
8. D
9. B
10. C
11. A
12. D
13. D
14. C
15. C
16. B

17. D
18. A
19. A
20. C
21. B
22. C
23. C
24. B
25. C
26. A
27. C
28. B
29. D
30. A
31. A
32. B
33. A

34. D
35. B
36. A
37. D
38. A
39. B
40. D

CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10
PART II

April 16th, 2011

1. Simplify (assuming that $x \neq -\frac{3}{2}$)

$$\frac{2x^2 - 7x - 15}{2x^3 + 13x^2 + 15x}.$$

(a) $\dfrac{1}{30}$

(b) $\dfrac{x-5}{x+5}$

(c) $\dfrac{2x-3}{x(2x+5)}$

(d) $\dfrac{x-5}{x(x+5)}$

2. A basketball team had a ratio of wins to losses of 3:1. After the team won six games in a row, its ratio of wins to losses became 5:1. How many games had the team won **before** winning six games in a row?

(a) 3
(b) 6
(c) 9
(d) 54

3. $ABCD$ is a rectangle whose vertices are at $A(2,0)$, $B(0,3)$, $C(8,4)$, and $D(6,7)$. If the diagonals of $ABCD$ intersect at E, what are the coordinates of E?

(a) (2, 3.5)
(b) (3, 5)
(c) (4, 0.5)
(d) (4, 3.5)

4. S is the set of all positive numbers n such that $1 < n < 100$ and $\sqrt{n}$ is an integer. What is the average of the members of set S?

(a) 25
(b) 35.5
(c) 40
(d) 50

5. Where (from the left) will a 12 ft. beam balance if a 125 lb. weight is placed on the right end and a 75 lb. weight is placed on the left end?

(a) 5 feet
(b) 6 feet
(c) 7.5 feet
(d) 8 feet

6. Eleven teams play in a soccer tournament. Each team must play each of the other teams exactly once. If a game ends in a tie, each team gets 1 point. For the games that do not end in a tie, the winning team gets 5 points and the losing team gets 0 points. Which of the following is a possible value for the total number of points earned by the 11 teams by the end of the tournament?

(a) 290
(b) 257
(c) 196
(d) 92

7. What is the remainder when 12345678910111213 is divided by 999?

(a) 0
(b) 13
(c) 271
(d) 523

8. Let a, b, c, d, and e be distinct integers such that

$$(6 - a)(6 - b)(6 - c)(6 - d)(6 - e) = 45.$$

What is $a + b + c + d + e$?

(a) 17
(b) 25
(c) 27
(d) 30

9. Triangle ABC has a right angle at B. Point D is the foot of the altitude from B, $AD = 3$, and $DC = 4$. What is the area of $\triangle ABC$?

(a) 21
(b) $14\sqrt{3}$
(c) $4\sqrt{3}$
(d) $7\sqrt{3}$

10. At Pine Summer Camp, 60% of the children play soccer, 30% of the children swim, and 40% of the children who play soccer swim. To the nearest whole percent, what percent of the non-swimmers play soccer?

(a) 51%
(b) 49%
(c) 45%
(d) 40%

11. What number is one third of the way from $\dfrac{1}{4}$ to $\dfrac{3}{4}$?

(a) $\dfrac{1}{3}$

(b) $\dfrac{5}{12}$

(c) $\dfrac{1}{2}$

(d) $\dfrac{7}{12}$

12. What is the largest possible integer n that satisfies $n^{200} < 3^{500}$?

(a) 13
(b) 14
(c) 15
(d) 16

13. How many values of x satisfy

$$\frac{2x^2 - 10x}{x^2 - 5x} = x - 3?$$

(a) 0
(b) 1
(c) 2
(d) 3

14. The sides of a triangle are 8, 13, and 15 inches. What is the area of the triangle?

(a) 52 square inches
(b) $20\sqrt{2}$ square inches
(c) 60 square inches
(d) $30\sqrt{3}$ square inches

15. In calm weather, an airplane can fly from one city to another 200 km north of the first and back in exactly 2 hours. In a steady north wind, the round trip takes 5 minutes longer. What is the speed of the wind?

(a) 8 km/h
(b) 16 km/h
(c) 32 km/h
(d) 40 km/h

16. For all real numbers a and b, which of the following is true?

(a) $a^2 + b^2 \leq 2ab$
(b) $a^2 + b^2 \geq 2ab$
(c) $a^2 + b^2 < 2ab$
(d) $a^2 + b^2 > 2ab$

17. Which of the following is a factor of $x^{50} - 2a^{47}x^3 + a^{50}$?

(a) $x - a$
(b) $x - a^2$
(c) $x + a$
(d) $x + a^2$

18. Let $f(x) = x^2 + Kx + 1$, where K is a real constant. If r and s are the real roots of $f(x)$, which of the following is impossible?

(a) $r = s$
(b) $rs = 1$
(c) $r + s = 1$
(d) $r > 0$ and $s > 0$

19. A gear of radius 1 revolves around a fixed gear of radius 2. During one complete revolution, the smaller gear will rotate

(a) $360°$
(b) $540°$
(c) $720°$
(d) $900°$

20. A three-digit number *abc* is chosen. The difference between *bca* and *abc* is calculated to lie between 400 and 500. The number *abc* =?

(a) 400
(b) 404
(c) 429
(d) 495

21. A triangle with sides of length 5, 7, and 9 is

(a) acute
(b) obtuse
(c) right
(d) impossible

22. Determine

$$\left(\sqrt{3}^{\sqrt{3}}\right)^{\sqrt{3}} \cdot \sqrt{3}.$$

(a) 1
(b) 3
(c) 9
(d) 27

23. A pentagon *MNPQR* has $MN = 2$, $NP = 7$, $PQ = 4$, $QR = 5$, and $RM = 1$. The sum of the lengths of the diagonals, $MP + MQ + NQ + NR + PR$, cannot possibly equal

(a) 53
(b) 33
(c) 18
(d) 16

24. Find all solutions of

$$x\sqrt{5 - x^2} = 2.$$

(a) ± 1
(b) ± 2
(c) $1, 2$
(d) both (a) and (b) are solutions

25. Rose and Mary play a series of three games. In each game, Rose's probability of winning is $\frac{2}{3}$. If Rose's probability of winning at least two of the three games is p, then

(a) $p < \frac{1}{2}$

(b) $\frac{1}{2} \le p < \frac{2}{3}$

(c) $p = \frac{2}{3}$

(d) $\frac{2}{3} < p \le \frac{3}{4}$

26. A square and an equilateral triangle have equal perimeters. The area of the triangle is $9\sqrt{3}$ in. How long is the diagonal of the square?

(a) $\frac{9}{2}$

(b) $\frac{9\sqrt{2}}{2}$

(c) $2\sqrt{5}$
(d) $4\sqrt{2}$

27. If a is a nonzero integer and b is a positive number such that $ab^2 = \log_{10} b$, what is the median of the set $\left\{0, 1, a, b, \frac{1}{b}\right\}$?

(a) 1
(b) a
(c) $\frac{1}{b}$
(d) b

28. An integer x, where $10 \le x \le 99$, is to be chosen. If all choices are equally likely, what is the probability that at least one digit of x is a 7?

(a) $\dfrac{1}{9}$

(b) $\dfrac{1}{5}$

(c) $\dfrac{19}{90}$

(d) $\dfrac{2}{9}$

29. Define the operation $\star$ by

$$a \star b = (a + b)b.$$

What is the value of $(3 \star 5) - (5 \star 3)$?

(a) 16
(b) 8
(c) 0
(d) -8

30. A chemist has two solutions of hydrochloric acid. One is a 40% solution and the other is a 90% solution. How many liters of each solution should she mix to get 10 liters of a 50% solution?

(a) 8 liters of 40% solution, 2 liters of 90% solution
(b) 2 liters of 40% solution, 8 liters of 90% solution
(c) 7 liters of 40% solution, 3 liters of 90% solution
(d) 3 liters of 40% solution, 7 liters of 90% solution

31. The number N is formed by writing all of the two digit numbers from 18 to 95 consecutively, i.e., $N = 181920...92939495$. What is the largest power of 3 that is a factor of N?

(a) 81
(b) 27
(c) 9
(d) 3

32. What is the value of 1 - 2 + 3 - 4 + ... + 99 - 100?

(a) -1
(b) 1
(c) -50
(d) 50

33. When the last digit of a certain six-digit number N is transferred to the first position, and the other digits are moved one place to the right, the new number is exactly one third of N. What is the sum of the six digits?

(a) 28
(b) 27
(c) 26
(d) 25

34. How many solutions of $2^{2x} - 2^{2y} = 55$ are there where x and y are integers?

(a) 0
(b) 1
(c) 2
(d) 3

35. If x is inversely proportional to y, y is inversely proportional to z, and z is inversely proportional to v, which of the following best expresses the relationship between x and v? Assume k is a constant.

(a) $x = \dfrac{k}{v}$
(b) $x = \dfrac{k}{v^2}$
(c) $x = kv$
(d) $x = kv^2$

36. A regular polygon of 300 sides, F_1, is inscribed in a circle. Inscribed in the same circle is another regular polygon, F_2, with 600 sides. The perimeters P_1 and P_2 of F_1 and F_2, respectively, satisfy

(a) $P_1 = P_2$
(b) $P_1 < P_2$
(c) $P_1 > P_2$
(d) $P_1 + P_2 = 2P_1 - P_2$

37. A bird in the hand is worth two in the bush. Five birds in the hand and three birds in the bush are worth $15 more than three birds in the hand and five birds in the bush. A bird in the hand is worth

(a) $5.00
(b) $7.50
(c) $10.00
(d) $15.00

38. If $a = 6\sqrt{3} - 3\sqrt{13}$ and $b = 6\sqrt{10} - 15\sqrt{2}$, then

(a) $a - b \geq 1$
(b) $0 < a - b < 1$
(c) $-1 < a - b < 0$
(d) $a - b \leq -1$

39. A palindrome is a number that reads the same forwards and backwards, like 121. How many four-digit numbers are palindromes?

(a) 81
(b) 90
(c) 100
(d) 121

40. Determine n so that

$$1 + 2 + \ldots + n = (n+1) + (n+2) + \ldots + 118 + 119.$$

(a) $n = 60$
(b) $n = 69$
(c) $n = 76$
(d) $n = 84$