Functional Analysis and Mathematical Physics Interdepartmental Research Group (FAMP) Colloquium Series Fall 2020

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Talk 10: *An Introduction to Complex Dimensions: The Case of Fractal Strings*

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We provide an introduction to the mathematical theory of complex fractal dimensions (developed by the author and his collaborators), which captures the vibrations that are intrinsic to both fractal geometries and the prime numbers. We focus here on the case of fractal strings, or one-dimensional drums with fractal boundaries. Complex dimensions are the poles of suitably defined geometric zeta functions associated to fractal strings. Intuitively, their real and imaginary parts correspond respectively to the amplitudes and the frequencies of "geometric waves" traveling through the space of scales associated with the fractal string. Explicit formulas, significantly extending Riemann's original explicit formula for the prime number counting functional and the Riemann zeros, enable us to express very precisely the oscillations intrinsic to fractal and arithmetic geometries, via the underlying complex dimensions. Key examples of such formulas are fractal tube formulas and spectral asymptotic formulas with complex exponents, along with formulas for the prime orbit counting functions of certain dynamical systems generalizing the dynamical counterpart of the Prime Number Theorem. We will illustrate aspects of the theory by means of the Cantor string as well as via self-similar strings, the complex dimensions of which happen to exhibit generically very intriguing quasiperiodic patterns, which we conjecture to form (generalized) quasicrystals. In the next lecture, we plan to discuss the higher-dimensional theory of complex dimensions, and of the corresponding fractal zeta functions. The main reference for this talk is M. L. Lapidus and M. van Frankenhuijsen, Fractal Geometry, *Complex Dimensions and Zeta Functions,* Springer Monographs in Mathematics, Springer, New York, 2013 (second revised and enlarged edition).

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Friday, November 20, 2:00-3:30 PM (PST), Online via Zoom at
https://fresnostate.zoom.us/j/5233106532