# A Model of Interest Group Competition and Cooperation

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## Abstract

This appendix provides a more formalized model of the competitive model of interest group competition and the conditions under which coalitions may form described in my article "Interest Group Competition and Coalition Formation." It also explains how this model was transformed into a statistical model using the concept of quantal response equilibrium as laid out by McKelvey and Palfrey and developed by Signorino.

It is perhaps most useful to consider the more formalized, although not truly "formal" in a mathematical sense, model of strategic decision making by lobbyists in a state of competition laid out in the first section of this appendix as essentially replacing that part of the published paper following the section heading "A Model of Coalition Formation in a Competitive Environment." The second part of this appendix, however, does not directly replace any part of the published paper, although it expounds on those sections where I describe Signorino's strategic probit model in words. In other words, it is best to consider the section here titled "A Probabilistic Version of the Competitive Model" as additional material coming after the first paragraph in the "Research Design" section of the article. Figures 1 and 2 used here are the same as those in the published article.

## The Competitive Model

I conceive of the model as a game where each lobbyist will take the optimal position on an issue given expectations of the choices of competitors, as well as pressures from their own group members and legislators. Coalitions form, I assume, only if two or more lobbyists find it optimal to support the same issue position, otherwise they will expend resources in an advocacy conflict with each other. Let $\boldsymbol{X} = \{x_1, x_2, \ldots, x_i\}$ be a uni-dimensional continuum of positions representing possible policy solutions to some issue problem facing legislators. Advocating on this issue is a set of lobbyists $\boldsymbol{G} = \{1, 2, \ldots, k\}$ representing organized interest memberships, and $S_k$ a vector of strategies available to each lobbyist $k$, who must decide whether to support each position, in the set of all strategies $\boldsymbol{S}$ such that $\boldsymbol{S} = \boldsymbol{\Pi}_{k \in G}\, S_k$. Lobbyist preferences for strategies of support for, or rejection of, positions in $\boldsymbol{X}$ are represented by the continuous and concave utility function $u_k$: $S \rightarrow \mathfrak{R}$ so that lobbyist $k$'s strategy choice to support or oppose each position

$x_i$, designated $s_{ki}$, yields a payoff expressed in real numbers. Lobbyist $k$'s decision rule for each $s_{ki} \in S_k$ is

$$s_{ki} = \begin{cases} s_i^s, \text{ or support } x_i, \text{ iff } u_k(s_i) = \max u_k(s) \\ s_i^n, \text{ or not support } x_i, \text{ otherwise} \end{cases}$$

so that a lobbyist chooses a strategy supporting a position $x_i$, $s_{ki} = s_i^s$, only if $u_k(s_i^s) > u_k(s_{-i})$, given that if $s_{ki} = s_i^s$ then $s_{k\text{-}i} = s_{-i}^n$ $\forall s_{k\text{-}i} \in S_k$ (the lobbyist can only support one position on the issue).

For the moment I let each lobbyist be myopic with respect to the choices of competing lobbyists so that their payoffs for choosing $s_i^s$ or $s_i^n$ is simply the sum of the two utilities representing collective group member and legislator pressures:

$$u_k(s_i) = u_k(m_k) + u_k(l) \qquad (1)$$

To capture the constraining effect of each audience, let $M_k$ be the number of members in lobbyist $k$'s organization and $\boldsymbol{L}$ be the members of a legislature, all of whom have their own preferences over $\boldsymbol{X}$ normally distributed around mean positions $\bar{x}_{ki}$ and $\bar{x}_L$ respectively so that only by choosing to support these positions does $k$ receive the maximum level of utility from an audience. Consistent with other spatial models, deviating from these ideal positions costs the lobbyist utility per a quadratic loss function. That is, any strategy supporting a position other than $\bar{x}_{ki}$ reduces $u_k(m_k)$ by $M_k - \left[\theta_k(\bar{x}_{ki} - x_i)^2\right]$ where $(\bar{x}_{ki} - x_i)^2$ is the distance from the mean group ideal point to an alternative point and $\theta_k$ is the rate of member loss as this deviation increases, which I interpret as the intensity of preference members collectively have for their group ideal position. Lobbyists similarly lose support from legislators by $L - \left[\psi(\bar{x}_L - x_i)^2\right]$ as $(\bar{x}_L - x_i)^2$ increases with $\psi$ representing this rate of utility loss.

While I assume a single legislature, each lobbyist has his or her own set of group members. If each group's collective ideal point is the mean $\bar{x}_{ki}$, then competition exists when for two or more interest groups $\bar{x}_{ki} \neq \bar{x}_{-ki}$, the degree of competition thus being the ideological distance between them multiplied by their collective preference intensity values $\theta_k$ and $\theta_{-k}$. By pairing a group represented by $k$ once with every other group $-k$ lobbying the same issue, so that $c$ is one pair and $\boldsymbol{C_G}$ the set of all pair combinations in $\boldsymbol{G}$, the level of competition among groups on an issue can be calculated by the formula

$$\frac{\sum_{c=1}^{c \in C_G} \left[ \frac{[\theta_k (\bar{x}_{ki} - \bar{x}_{-ki})^2][\theta_{-k} (\bar{x}_{ki} - \bar{x}_{-ki})^2]}{(\bar{x}_{ki} - \bar{x}_{-ki})^2} \right]}{C_G} \tag{2}$$

The product of each pair is divided by ideological distance (so that it is not included twice) and all products are summed and then divided by the total number of group pairs to produce an average competition score comparable across issues.[1]

The influence of group members is assumed to be independent of legislators, so the utility function laid out in equation 1 is concave and decreasing around some maximum value.[2] Assuming that $\bar{x}_{ki} \neq \bar{x}_L$, and that $x_i$ is in the closed interval $[\bar{x}_{ki}, \bar{x}_L]$, the value of a strategy supporting a position in $\boldsymbol{X}$ to a lobbyist depends on a trade-off between group member and legislator utility. Therefore, as $(\bar{x}_{ki} - x_i)^2$ increases, $u_k(m_k)$ falls, and as long as $(\bar{x}_L - x_i)^2$ decreases, $u_k(l)$ rises in value and *may* compensate the lobbyist for the loss of group members.[3] The result is that there may be positions a lobbyist values over his or her group member ideal. Such an *alternate set* would contain points in $\boldsymbol{X}$ only if $u_k(s_i^s) > u_k(\bar{s}_i^s)$, where $\bar{s}_i^s$ is a strategy supporting $\bar{x}_{ki}$. Because the utility curves for $u_k(m_k)$ and $u_k(l)$ are decreasing around $\bar{x}_{ki}$ and $\bar{x}_L$, as $(\bar{x}_{ki} - x_i)^2$ increases this inequality holds only as long as the gain in legislator support (the derivative of $u_k(l)$) outweighs the loss from exiting group members. Figure 1 illustrates this trade-off for a lobbyist where the lines *dM* and *dL* represent the rates of gain and loss for strategies supporting positions in ***X*** with the lobbyist's alternate set *A* ranging from $\bar{x}_{ki}$ to a position $x_i$ where $u_k(s_i^s)=u_k(\bar{s}_i^s)$.[4] Both components of the utility function are assumed to increase and decrease at constant rates, so the position $x_{ki}^*$ where $u_k(s_i^*) = \max u_k(s)$ must be in the center of the alternate set and is the lobbyist's choice because that position alone provides a payoff greater than any other. If $\theta_k$ increases, the rate of group member loss increases from *dM* to *dM′* in Figure 1, and the range of positions in ***X*** the lobbyist values over $\bar{x}_{ki}$ falls to *A′*. More importantly, the ideological distance between the group member ideal position and the position the lobbyist actually chooses also decreases. Conversely, a decrease in $\theta_k$, or an increase in $\psi$, expands the range of the alternate set.

---- Insert Figure 1 about here ----

Now I can incorporate into lobbyist *k*'s calculation an expectation regarding the choices of competing lobbyist -*k* for whom $\bar{x}_{ki} \neq \bar{x}_{-ki}$.[5] If *k* and -*k* both have strategies they prefer to $\bar{s}_{ki}^s$ and $\bar{s}_{-ki}^s$, then there may also be one or more positions that they would *both support*, thus providing common ground for cooperation, coalition building, and resource sharing. But unless $x_{ki}^* = x_{-ki}^*$, simply having overlapping alternate sets is not sufficient to identify such positions. The utility for each lobbyist's strategy supporting a common position must also increase so that $u_k(s_i) > u_k(s_i^*)$ is true for all or a subset of lobbyists in ***G***. Hula (1999) finds that coalitions often form as a result of lobbyists offering each other incentives, such as financial and political resources. Assuming that a coalition can only support a single position $x_i$ on an issue, the inequality $u_k(s_i) > u_k(s_i^*)$ can only be true if lobbyist *k* gains additional resources, and therefore utility, for supporting a position other than $x_i^*$ from competitors who *also* choose to support that position. This incentive shaping $k$'s decision to support a position is therefore *conditional* on the expectation that competitor $-k$ will support that same position because resources will not be shared otherwise. There may even be several points in $\boldsymbol{X}$ where this inequality holds due to resource sharing among some subset of lobbyists.

To explore this let $\boldsymbol{R} = \{R_1, R_2, \ldots, R_j\}$ be the set of all possible combinations of lobbyists in $\boldsymbol{G}$ so that each $R_j$ is a profile of at least $k + 1$ lobbyists. Each possesses some amount of resources $r$ so that if $k \in R_j$, and $-k$ identifies all competing lobbyists in $R_j$, then

$$r_{j\text{-}k} = \sum_{-k=1}^{-k \in R_j} r_{-k}$$

is the amount of resources $k$ gains *if all* competitors in the profile chose to support a common position and therefore form a coalition with $k$. Let $r_{j\text{-}k}$ map on to $\Re$ by each lobbyist's utility function so that $k$'s gains for cooperating with the other lobbyists in $R_j$ is $u_k(r_{j\text{-}k})$ and the utility for a strategy supporting a joint position with them is now

$$v_k(s_{ji}^{s}) = u_k(s_i^{s}) + u_k(r_{j\text{-}k}) \qquad (3)$$

where $v_k(s_{ji}^{s}) > u_k(s_i^{*})$ only if all competing lobbyists in profile $R_j$ choose to form a coalition and share resources. Keep in mind that each profile $R_j$ is only a *potential* coalition, and a lobbyist may be in several profiles offering varying amounts of resources so that the value of $v_k(s_{ji}^{s})$ likely varies from profile to profile and $k$ must choose between them. Define $P_j = \{P_{j1}, P_{j2}, \ldots, P_{jk}\}$ where $P_{jk} \in \boldsymbol{X}$ is a vector of positions that any lobbyist $k$ in profile $R_j$ prefers to support over $x_{ki}^{*}$ only if resources are shared due to the increase in the value of strategies by $u_k(r_{j\text{-}k})$ in equation 3. A point $x_i$ is thus an element in $k$'s *preferred-to set* $P_{jk}$ for profile $R_j$ only if

$$v_k(s_{ji}^{s}) - u_k(s_i^{*}) > 0 \qquad (4a)$$

for the strategy associated with that position. Define $\boldsymbol{C} = \{C_1, C_2, \ldots, C_j\}$ as a *coalition set* associated with profile $R_j$ where $C_j$ is a set of points in $\boldsymbol{X}$ that are also elements in *all* of the preferred-to sets of all lobbyist in $P_j$ (meaning where all of the preferred sets associated with $R_j$ intersect). A coalition set is not empty, and therefore a coalition is *feasible*, and $x_i \in C_j$ if, for the associated strategies of all lobbyists in $R_j$, the condition

$$\sum_{k=1}^{k \in R_j} [v_k(s_{ji}^s) - u_k(s_i^*)] > 0 \qquad (4b)$$

holds, otherwise $C_j = \varnothing$ and the lobbyists in that profile will not form a coalition.[6]

If lobbyist $k$ is in more than one profile, and the coalition sets associated with those profiles contain positions he or she would prefer to support if resources were shared rather than lobby alone, then $k$'s choice regards which feasible coalition to join. To determine the attractiveness of one of these possible coalitions, I calculate their value by determining which position $x_i$ all members of profile $R_j$ would support if they actually did form a coalition. An analogy for coalition formation is bargaining among a set of actors who have more to gain by working together than by fighting each other. I therefore employ the bargaining model pioneered by Nash (1950; 1953) where the solution is a combination of individual strategies jointly providing a greater payoff to all actors than any other combination, including the one where no agreement is reached.[7] Define $S_{ji}^*$ as a set of strategies supporting position $x_i$ that all lobbyists in profile $R_j$ will choose only if

$$v_j(S_i^*) = \arg\max \prod_{k=1}^{k \in R_j} [v_k(s_{ji}^s) - u_k(s_i^*)] \qquad (5a)$$

is true. As Nash showed generally, the position associated with this set of strategies is the only one that is optimal for that profile and is therefore the only position a feasible coalition would support if it actually formed. The coalition value to each individual lobbyist for this profile is therefore

$$v_k(c_{ji}) = v_k(s^s_{ji}) - u_k(s^*_i) \tag{5b}$$

or simply how much more $k$ gains by supporting the coalition position over the optimal position in his or her alternate set. Equation 5b provides this value for every feasible coalition for every lobbyist which, in turn, makes it possible to determine which coalition (if any) a lobbyist will actually choose to join. Because the coalition value of a strategy supporting position $x_i$ is conditioned on every other lobbyist in $R_j$ likewise choosing to support it and share resources, $k$'s choice is characterized as a best response. Specifically, $k$ seeks to maximize $v_k(c)$ over all feasible coalitions associated with profiles of which $k$ is a member, but this depends on the coalition values $v_{-k}(c)$ of all competitors in $R_j$ *also* being greater here than for any other feasible coalition so that they will make the same choices, or that $v(C^*_{ji}) = \max v_k(c)\ \forall k \in R_j$. Feasible coalition $C_j$ supporting $x_{ji}$ therefore only forms when it is an equilibrium outcome, or when

$$v_k(c_{ji}) > v_k(c_{-ji}) \tag{6}$$

is true for every lobbyist in $R_j$.

This inequality defining the equilibrium requires all lobbyists in profile $R_j$ to maximize their expected utility over all possible coalitions, thus making equilibrium outcomes sensitive to changes in $\theta$, $\psi$, and $r_{j\text{-}k}$. One consequence of this framework is that a lobbyist's choice is not dependent on a pre-existing status quo position because the status quo is itself merely a reflection of the preferences of legislators, group members, and competing lobbyists. If preferences remain stable, then any position garnering enough support to become law will endure as a status quo. Put another way, as Baumgartner and Jones (1993) argue, a status quo reflects unchanging preferences among decision makers, often because the set of players itself tends to remain unchanged, so that there is no change of value in the terms of equation 3 that would entice or compel a lobbyist to prefer an alternate position. Thus the status quo simply remains the optimal choice and an equilibrium outcome. A "punctuated equilibrium," where a status quo is overturned, would result when a change in pressure compels lobbyists to jointly value, via equation 6, another position over the old status quo or if the set of lobbyists in ***G*** is significantly changed. The status quo is therefore a consequence of the model's inputs; it is not an exogenous input itself.

Given this framework, I can state a few propositions regarding competition and coalition formation among competing interest group lobbyists:

*Proposition 1: The more resources $r_k$ lobbyist k offers, the greater the likelihood k joins a coalition supporting a position minimizing $(x^*_{ji} - x^*_{ki})^2$, closer to k's non-coalition optimal position.* My explanation proceeds in two steps. First, the tension between the incentive $u_k(r_{j\text{-}k})$ to support a coalition position $x_{ji}$ and pressure from group members to support $\bar{x}_{ki}$ can be expressed as a ratio giving the range of $k$'s preferred set for profile $R_j$, $P_{jk}$, or $dr_{j\text{-}k} / d\theta_k$. Because the denominator represents the rate of utility loss as members exit, as $\theta_k$ increases the denominator increases thus reducing the number of possible positions in $P_{jk}$ as well as the distance between $\bar{x}_{ki}$ and $x^*_{ki}$. Alternatively, if $k$ increases the resources he or she is dedicating to advocacy, then $u_{\text{-}k}(r_{jk})$ must increase for all competitors in all profiles containing $k$. This means that the range of all competitors' preferred sets must increase at a constant rate because $r_k$ is the same across all profiles containing $k$. This, in turn, makes it more likely that the condition in equation 4b is fulfilled and lobbyist $k$ faces fewer empty coalition sets and therefore has more feasible coalitions from which to choose.

To clarify the second part, I graph in Figure 2 the utility curves for lobbyist $k$ and one competitor, designated 1 and 2, who compose profile $R_j$. The solid curves represent each lobbyist's utility per equation 3, including utility from the resources he or she anticipates sharing with the other, where the value of strategies are maximized over positions $x^*_{1i}$ and $x^*_{2i}$ respectively. All strategies associated with positions yielding a utility higher than $u_1( s^*_i )$ and $u_2( s^*_i )$ on each vertical axis are now preferred to the non-coalition optimal positions $x^*_{1i}$ and $x^*_{2i}$, as indicated by the vertical dashed lines starting where the horizontal lines intersect the utility curves. As seen below the ***X***-continuum, these define the preferred sets $P_{j1}$ and $P_{j2}$ for each lobbyist and where these sets intersect is the coalition set $C_j$. Since $C_j \neq \varnothing$, if 2 increases resources to $r_2'$, then the utility 1 receives for a support strategy increases by $u_1(r_{j2})'$. Although this does not change the shape of 1's utility curve (the rate of gain and loss remains constant), what *does* change per equation 4a is the range of $P_{j1}$ to $P_{j1}'$, as indicated by the dashed curve in Figure 2, and, as a consequence of equation 4b, the range of the coalition set, now designated as $C_j'$. Because it is 1's utility curve that is rising, $C_j'$ expands towards 2's non-coalition optimal point $x^*_{2i}$, which means that, because the joint strategy $v_j( S^*_i )$ must support a coalition position $x^*_{ji}$ in the center of $C_j$, the new coalition position $x^{*'}_{ji}$ must also have shifted right.[8] Reducing $(x^*_{2i} - x^{*'}_{ji})^2$ means that $v_2(s^*_{ji})' - u_2(s^*_i)$ also decreases so that the position lobbyist 2 will support is closer to his or her group member ideal than it would be at lower values of $r_2$. The substantive consequence is that large, wealthy organizations, perhaps "peak" associations representing powerful social or economic groups, are more likely to assemble ideologically broad coalitions and have policy problems resolved in their favor.[9]

---- Insert Figure 2 about here ----

*Proposition 2: The greater the range of ideological positions in $C_j$, the smaller the difference $v_k(s^*_{ji}) - u_k(s^*_i)$ for lobbyists with non-coalition optimal positions closer to the legislative mean.* Recall that as $\theta_k$ decreases, or $u_k(r_{j-k})$ increases, the range of $P_{jk}$ increases for lobbyist $k$. As shown for Proposition 1 this causes coalition set $C_j$ to expand which, in turn, means that coalitions will be ideologically broad and thus more likely to contain $\bar{x}_L$. Because the position representing the Nash solution, $x_{ji}$, is in the center of $C_j$, as $P_{jk}$ increases when $k$ is on the opposite side of $\bar{x}_L$ from $-k$, then $(\bar{x}_L - x_{ji})^2$ must decrease. The consequence of this coalition set expansion is that lobbyists with optimal positions closer to the legislative mean (and median since I assume legislator preferences are Normally distributed), arguably those representing ideologically moderate interests, will see a decrease in $(x_{ji} - x^*_{ki})^2$. This means that the distance $v_k(s^*_{ji}) - u_k(s^*_i)$ also decreases so that the coalitions these lobbyists choose to support are more likely to produce joint positions closer to both their optimal and group member ideal positions than are lobbyists for more ideologically extreme interest groups. In fact, the latter are less likely to join a coalition at all unless they are under tremendous pressure from legislators (very high values of $\psi$), which suggests a bias in coalition formation favoring ideologically moderate interest groups.

*Proposition 3: Lobbyists representing members who are more ideologically extreme, or high values of $(\bar{x}_{ki} - \bar{x}_L)^2$, and feel intensely about an issue are less likely to join a coalition.* To see this recall, that equation 4a does not change the size of lobbyist $k$'s alternate set if he or she chooses to join a feasible coalition because the re-valued payoff for strategies supporting all points in $\boldsymbol{X}$, including $\bar{x}_{ik}$ and $x^*_{ki}$, is increased by $u_k(r_{j-k})$ which is a positive constant. As long as $u_k(r_{j-k}) > [u_k(\bar{s}^s_i) - u_k(s^*_i)]$, a lobbyist's preferred-to set for profile $R_j$ will be larger than, and encompass all of, his or her alternate set. A non-empty coalition set is an intersection of the preferred sets of $k + 1$ lobbyists in $R_j$ so that there exist points that all lobbyists in the profile value over their optimal strategy $s_{ki}^{*}$ for advocating alone, but the range of positions in $C_j$ is sensitive to changes in the parameter $\theta$ for any $k \in R_j$. If $\theta_k$ increases, so that $\theta_k < \theta_k'$, the rate of group member loss to $k$ for supporting $x_i \mid x_i \neq \bar{x}_{ki}$ must increase and the range of positions in $k$'s alternate set decreases. Because $\theta$ is also part of $v_k(s_{ji}^{s})$, and $u_k(r_{j\text{-}k})$ is a positive constant in equation 3, by equation 4a the range of $k$'s preferred set must decrease as well, leading, by equation 4b, to a decrease in the range of positions in $C_j$ as well. In other words, any decrease in preferred positions for any one member must decrease the range of points that all members would prefer to a strategy of lobbying alone showing that there is an inverse relationship so that if $\theta_k < \theta_k'$ then $P_{jk} > P_{jk}'$ and $C_j > C_j'$. Conversely, an increase in $\psi$, which affects all lobbyists, has the opposite result as long as $x_i$ lies in the interval $[\bar{x}_{ki}, \bar{x}_L]$ for all $k \in R_j$. Finally, a rise in the value of $u_k(r_{j\text{-}k})$ increases the difference $v_k(s_{ji}^{*}) - u_k(s_i^{*})$ for all points in $\boldsymbol{X}$, thus expanding the range of positions in $C_j$ and making it less likely to be empty.

### A Statistical Version of the Competitive Model

A major obstacle to empirically testing the competitive model is designing a statistical model capable of capturing the interdependence of lobbyists' strategic choices. Modeling strategic interaction statistically has recently become a subject of great interest in the social sciences. Drawing on the random utility and discreet choice models literature where dependent variables are estimated as functions of known utility and randomly distributed error terms (e.g., McFadden 1974), McKelvey and Palfrey (1995; 1996; 1998) developed a quantal response equilibrium (QRE) solution for extensive form games. Unlike traditional deterministic game theory models, QRE solutions are probabilistic due to the inclusion of some unknown element in each player's expected utility calculation so that they can only anticipate the choices of others with some known probability distribution analogous to the random error term in standard statistical models. But in QRE models the disturbance term is assumed to have a theoretical justification, often some variation on notions of bounded rationality, thus taking a step towards bridging the gap between deterministic theoretical and probabilistic statistical models. Although QRE was developed in the context of experimental research, Signorino (1999) demonstrates how probabilistic theoretical models of strategic interaction can be estimated using maximum-likelihood estimation methods using real world data, although the exact form of the statistical model depends on the theoretical explanation behind the disturbance term (Signorino 2003). Equation 3 is thus re-written as

$$v_k( s^s_{ji} ) = \hat{v}_k( s^s_{ji} ) + \pi_k \tag{7}$$

where $\hat{v}_k( s^s_{ji} )$ is the known payoff for supporting $x_i$, including the resources $k$ expects to receive from $-k$, and $\pi_k$ represents $k$'s uncertainty regarding how likely the competitor is to actually support $x_i$ and share resources. The likelihood of $k$ choosing to support a position is therefore a function of a set of independent variables representing his or her known utility plus uncertainty captured by a Normal probability distribution over $-k$'s choice so that whether $k$'s choice is a best response is only known probabilistically.

To create a statistical model reflecting this I restrict the model to two competing lobbyists, $\boldsymbol{G} = \{1,2\}$, where each selects strategy $s^s_i$ to support a position $x_i \in \boldsymbol{X}$ or strategy $s^n_i$ of opposition. The subscript $k$ is no longer required as lobbyists are simply identified by the subscripts *1* and *2*. Because one lobbyist's choice is a best response to the anticipated choice of the other, I model lobbyist 2 as choosing *after* observing 1's choice so that he or she is making a binary choice under one of two circumstances: where lobbyist 1 has chosen $s^s$ or where he or she has chosen $s^n$. This results in four possible equilibrium outcomes, designated $y_s$, depending on the binary choices of both lobbyists given their utility functions from equation 3:

$$y_s = \begin{cases} y_1 = s^s, s^s \text{ if } v_1(s^s) > v_1(s^n) \text{ and } v_2(s^s) > v_2(s^n) \\ y_2 = s^s, s^n \text{ if } v_1(s^s) > v_1(s^n) \text{ and } v_2(s^s) < v_2(s^n) \\ y_3 = s^n, s^s \text{ if } v_1(s^s) < v_1(s^n) \text{ and } v_2(s^s) > v_2(s^n) \\ y_4 = s^n, s^n \text{ if } v_1(s^s) < v_1(s^n) \text{ and } v_2(s^s) < v_2(s^n) \end{cases} \qquad (8)$$

Although I am interesting in estimating the choices the lobbyists make rather than the outcomes they reached as a consequence of these choices, each choice is contingent on the utility each expects to receive given the probability of each of these equilibrium outcomes occurring. Per equation 7, each lobbyist's utility for a strategy leading to an outcome has a known and an unknown component. The known is the utility the lobbyist anticipates receiving for a particular outcome, and is therefore a function of a vector of independent variables and effects parameters, $\hat{v}_1(s^s_{ji})$ becoming $\hat{v}^s_1 = \beta_1 X_1$ for lobbyist 1, allowing me to re-write equation 7 as:

$$v^s_1 = \beta_1 X_1 + \pi_1 \qquad (9)$$

The disturbance term $\pi$ represents lobbyist 1's uncertainty regarding how intensely the competitor's group members feel about their mean ideal position on the issue, and therefore the level of flexibility the competitor has to choose alternative positions in ***X***. All he or she knows is that there is some commonly known average value of a distribution such that $\pi \sim N(0,\sigma^2)$.

As seen in equation 8, lobbyist 2's choice is a best response selecting outcome $y_1$ or $y_2$ if lobbyist 1 chose $s^s$, and outcome $y_3$ or $y_4$ if lobbyist 1 chose $s^n$. The probability that lobbyist 2 will choose *s* given that lobbyist 1 has done so, or $p^s_2 s$, is, following equation 8 above, that Pr[$v_2(y_1) > v_2(y_2)$], which becomes Pr[$\hat{v}_2(y_1)+\pi_{21} > \hat{v}_2(y_2)+\pi_{22}$] after adding the disturbance term. Re-arranging the terms leads to a probabilistic equation that can be used in a likelihood function:

$$p_2^s s = \Pr[\pi_{22} - \pi_{21} < \hat{v}_2(y_1) - \hat{v}_2(y_2)]$$

$$= \Phi\left[\frac{\hat{v}_2(y_1) - \hat{v}_2(y_2)}{\sqrt{\sigma^2_{\pi_{22}} + \sigma^2_{\pi_{21}}}}\right] \qquad (10)$$

where $\Phi$ is the standard normal cumulative distribution with variance $\sigma^2$. The probability of lobbyist 2 choosing $s^n$, given that 1 has chosen $s^s$, is $1 - p_2^s s$ and that 2 chooses $s^s$ given that 1 chooses $s^n$ is

$$p_2^n s = \Phi\left[\frac{\hat{v}(y_3) - \hat{v}(y_4)}{\sqrt{\sigma^2_{\pi_{24}} + \sigma^2_{\pi_{23}}}}\right] \qquad (11)$$

Lobbyist 1's expected utility for choosing $s^s$ or $s^n$ also depends on the probability of particular outcomes being reached, but this, in turn, depends on the probabilities of 2 making certain decisions. The probability that 1 chooses $s^s$, or $p_1^s s$, therefore combines the probabilities of 2's choice so that $\Pr[v_1(s^s) > v_1(s^n)]$ becomes

$$p_1^s s = \Pr[\hat{v}_1(s^s) + \pi_1 > \hat{v}_1(s^n) + \pi_1]$$

$$= \Pr[\,(p_2^s s\hat{v}_1(y_1) + \pi_{11} + (1 - p_2^s s)\hat{v}_1(y_2) + \pi_{12}) > (p_2^n s\hat{v}_1(y_3) + \pi_{13} + (1 - p_2^n s)\hat{v}_1(y_4) + \pi_{14})\,]$$

$$= \Pr[\,(p_2^n s\pi_{13} + (1 - p_2^n s)\pi_{14}) - (p_2^s s\pi_{11} + (1 - p_2^s s)\pi_{12}) <$$

$$(p_2^s s\hat{v}_1(y_1) + (1 - p_2^s s)\hat{v}_1(y_2)) - (p_2^n s\hat{v}_1(y_3) + (1 - p_2^n s)\hat{v}_1(y_4))\,]$$

$$= \Phi\left[\frac{(p_2^s s\hat{v}_1(y_1) + (1 - p_2^s s)\hat{v}_1(y_2)) - (p_2^n s\hat{v}_1(y_3) + (1 - p_2^n s)\hat{v}_1(y_4))}{\sqrt{(p_2^n s)^2\sigma^2_{\pi_{13}} + (1 - p_2^n s)^2\sigma^2_{\pi_{14}} + (p_2^s s)^2\sigma^2_{\pi_{11}} + (1 - p_2^s s)^2\sigma^2_{\pi_{12}}}}\right] \qquad (12)$$

As the probability that 1 chooses $s^n$ is simply $1 - p_1^n s$, a likelihood equation can be constructed estimating 1's choice as a function of 1's utility incorporating probabilities of 2's choice

$$L = \prod_{k=1}^{G} p_1^s s^{\delta_s} p_1^n s^{\delta_n} \qquad (13)$$

where $\delta_s$ is 1 if action $s$ was observed in the data and 0 otherwise, and $\delta_n$ is 1 if action $s^n$ was observed. This provides the general form of the strategic probit model used in the published paper's analysis.

**Figure 1**
**Change in a Lobbyist's Alternate Set and Optimal Strategy as Group Member Intensity Increases**

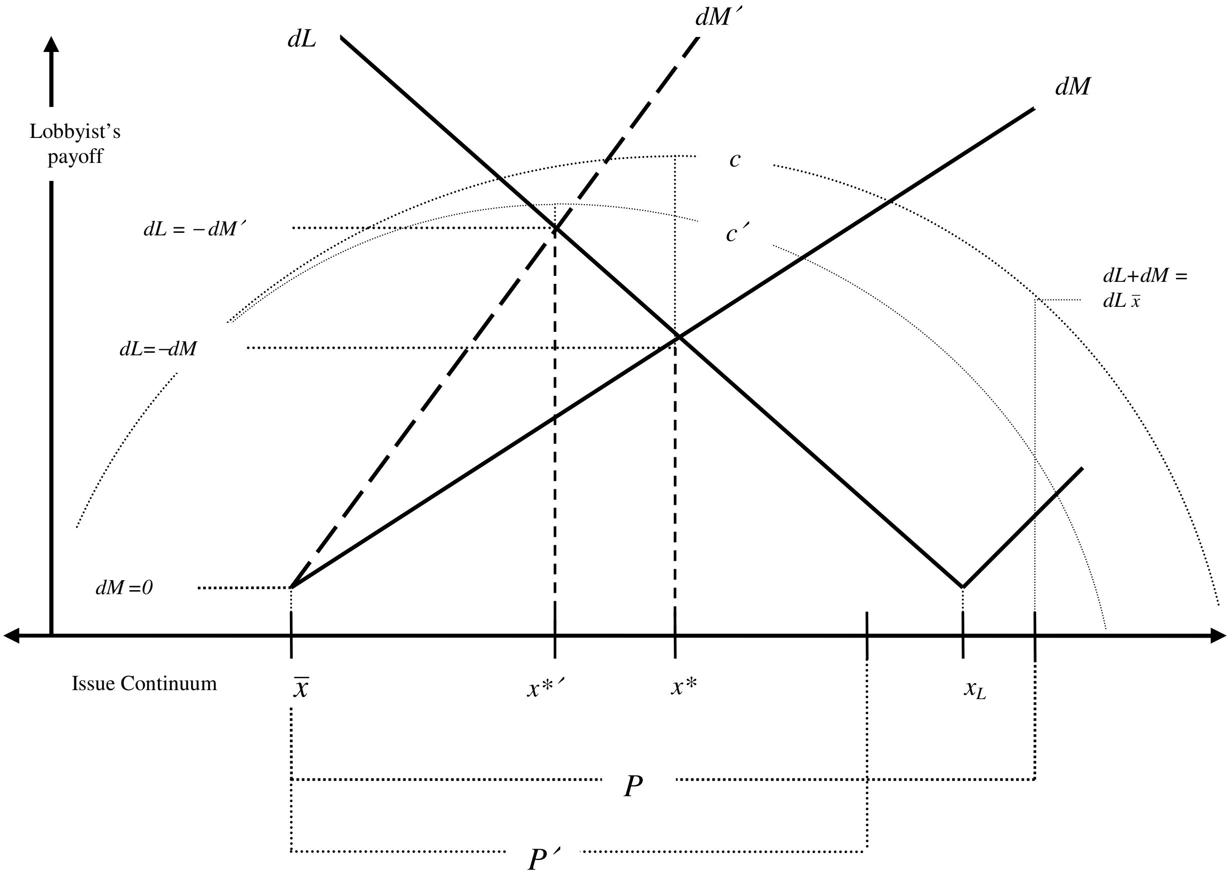
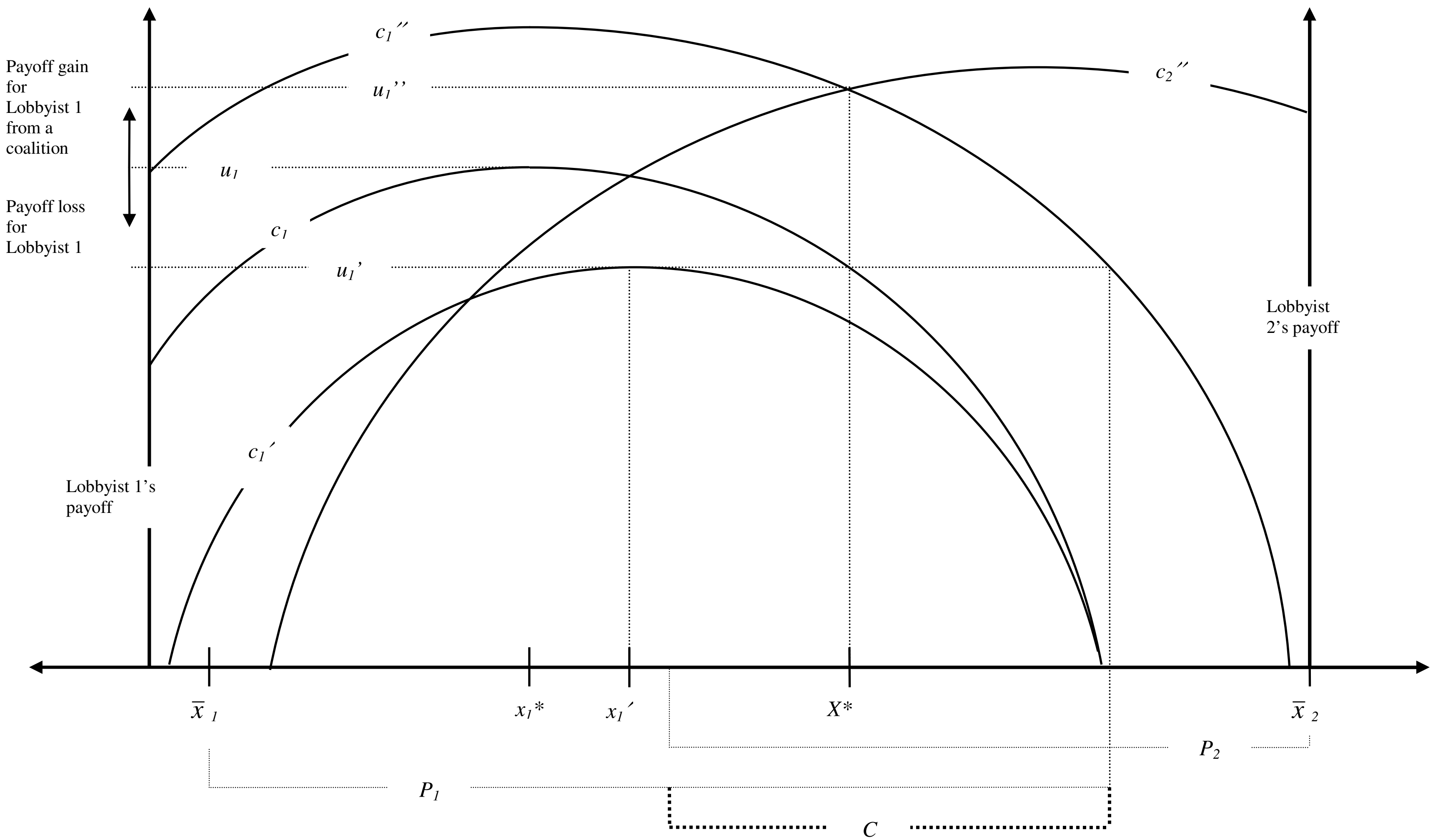


**Figure 2**
**Identification of Coalition Position due to Sharing Competitor Resources**



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[1] The number of group pairs $c$ is calculated using the combination formula $\boldsymbol{C_G} = \frac{G!}{(G-2)!2!}$ where 2 is used because I am interested in the number of interest group pairs.

[2] These functions are assumed to be symmetrical, or single-peaked, in that they fall away at the same rate regardless of direction from some maximum value.

[3] Note that the lobbyist's utility for a choice is not the actual gains or losses per se to either audience for a policy outcome at $x_i$, but how acceptable the strategy is to each audience.

[4] This is where the rate of group member loss plus the rate of gain in legislator support equals just the rate of gain in legislator support for a strategy supporting $\bar{x}_{ki}$ where $dM = 0$.

[5] The reader should clearly understand that if $\bar{x}_{ki} = \bar{x}_{-ki}$ the lobbyists would not be competing.

[6] Though all $x_i \in C_j$ must also be in $P_{jk}$ for all lobbyists in $R_j$, the reverse is not necessarily true.

[7] I employed the Nash solution over non-cooperative bargaining solutions, such as Rubinstein's (1982) discounting model, because the latter assumes some institutional framework dictating the structure and sequence of bargaining. Because it is unclear what institutional structure, if any, shapes bargaining among lobbyists, I used Nash's more general axiomatic approach, though work by Binmore et al. (1987) suggests that these models often come to the same solution.

[8] In the well-known Divide-the-Dollar Game, all things being equal, two bargainers will divide a dollar equally. The outcome only changes if one player's level of risk increases (Osborne and Rubinstein 1990) or if bargaining power becomes asymmetrical (Roth 1979).

[9] This should not be interpreted to mean that coalitions decrease in value to competitors. The corresponding increase in $v_{-k}( s^{s}_{ji} ) - u_{-k}( s^{*}_{i} )$ is compensated by rising values of $u_k(r_{j\text{-}k})$ so that the coalition value, and therefore the likelihood of $C_j$ being an equilibrium choice, remains the same.