# 2020-21 AY Assessment Report
# M.S. in Mathematics.

**Department and Program:** Department of Mathematics. Masters of Science in Mathematics
**Assessment Coordinator for the Graduate Program:** Oscar Vega.

1. **Learning outcomes assessed this year**

In the Fall of 2019, we transitioned into an M.S. in Mathematics from our former M.A. in mathematics, In the process, we eliminated an option, and so a new SOAP was written by the Graduate Committee. The assessment done in the last two years follows this new SOAP. We mention this now because there was no assessment report for the 2019-20 AY. In this report we only assess the 2020-21 AY.

Obviously, the courses and activities assessed in this report ran during the time Fresno State was closed because of the COVID pandemic. Hence, this report is not as complete as the one submitted two years ago, for example. Our plan is to come back to regular assessment activities during the 2021-22 AY.

## DIRECT MEASURES.

*(i) The first direct measure* is a summative assessment of MATH 200, MATH 228, and MATH 251. This is done by:

(a) Embedded questions on exams, midterms and/or finals; and/or

(b) Using a rubric to assess projects in classes that do not offer final exams. Rubrics are provided at the end of this document.

**(a) Embedded questions on exams, midterms and/or finals.**
The goals and SLOs assessed this year by using embedding of questions were:

**Goal A. Knowledge of Mathematics.** Students will acquire advanced knowledge in pure and applied mathematics, and in mathematics education, at the graduate level.

**Student Learning Outcomes.** Graduates will be able to:

**A1.** learn and understand advanced concepts in abstract algebra, analysis, and other pure mathematics topics.

**Goal B. Communicating Mathematics.** Students will continue learning to read, understand, and write rigorous mathematical proofs and other academic arguments, which they will communicate orally and in writing.

**Student Learning Outcomes.** Graduates will be able to:

**B1.** reconstruct proofs of classical theorems and/or write rigorous, multi-concept proofs using advanced concepts in pure mathematics, such as transformation, Lebesgue integral, uniform convergence, matrix groups, topological spaces, algebraic structures, etc.

**Goal C. Applications of Mathematics.** Students will apply mathematical knowledge to solve theoretical and practical problems in pure and applied mathematics, and in mathematics education.

**Student Learning Outcomes:** Graduates will be able to:

**C1.** apply the structural relationships among the various advanced concepts in pure mathematics to solve problems.

**C2.** use their knowledge of mathematics to examine new situations, analyze problems, and interpret their results.

**(b) Using a rubric to assess projects in classes that do not offer final exams.**
The goal and SLO assessed this year by using rubrics was:

**Goal A. Knowledge of Mathematics.** Students will acquire advanced knowledge in pure and applied mathematics, and in mathematics education, at the graduate level.

**Student Learning Outcomes.** Graduates will be able to:

**A3.** learn mathematics education theories and research methodologies.

**Goal B. Communicating Mathematics.** Students will continue learning to read, understand, and write rigorous mathematical proofs and other academic arguments, which they will communicate orally and in writing.

**Student Learning Outcomes.** Graduates will be able to:

**B3.** communicate the relationships among mathematics education theories, the gaps in the current literature, and what methods would be most applicable to investigating particulate mathematics education topics.

**B4.** use technology in written reports and oral presentations.

**Goal C. Applications of Mathematics.** Students will apply mathematical knowledge to solve theoretical and practical problems in pure and applied mathematics, and in mathematics education.

**Student Learning Outcomes:** Graduates will be able to:

**C3.** use their knowledge of mathematics education theories and research methods to plan research studies in mathematics education that would address gaps in the literature.

*(ii) The second direct measure* is to collect data based on our department rubric for each Project (MATH 298). Each member of a project committee submits a rubric on the writing and quality of work of the Project and on the quality of the defense.

*(iii) The third direct measure* is to collect data based on our department rubric for each Thesis (MATH 299). Each member of a thesis committee submits a rubric on the writing and quality of work of the Thesis and on the quality of the defense.

**INDIRECT MEASURES.**

We collected exit surveys and held exit interviews with the 2020-21 graduating class.

2. **Assignments/surveys used to assess the SLOs and criteria/rubrics used for evaluation.**

**(a) Embedded questions on exams, midterms and/or finals.**

This year, we embedded questions in midterms and final exams of MATH 228 and MATH 251. Also, given that some faculty (e.g. MATH 251 in this assessment report) write exams that allow students to not have to approach every question in the test, we decided that, when possible, we would embed more than one question per exam. In this case, we will take the best of the two scores for the same SLO a student obtained. Finally, some questions qualify for one, or two SLOs (never more than two); we will double count these questions (and their scores) separately for each SLO.

Exam 2 in MATH 228, exam 3 in MATH 251, and the final in MATH 228 were in-class only. The final exam in MATH 251 is take-home only.

We will use a universal expectation for all embedded questions: every student in the class should score at least 7/10 (70%) on every question.

**A1. Three embedded questions on the MATH 228 final exam in Spring of 2021.**
**One embedded question on the MATH 251 exam 3 in Fall of 2020.**
**One embedded question on the MATH 251 final exam in Fall of 2020.**

**B1. One embedded question on the MATH 228 exam 2 in Fall of 2020.**
**Two embedded questions on the MATH 228 final exam in Spring of 2021.**
**One embedded question on the MATH 251 exam 3 in Fall of 2020.**
**One embedded question on the MATH 251 final exam in Fall of 2020.**

**C1. Three embedded questions on the MATH 228 exam 2 in Fall of 2020.**
**One embedded question on the MATH 228 final exam in Spring of 2021.**

**One embedded question on the MATH 251 exam 3 in Fall of 2020.**
**One embedded question on the MATH 251 final exam in Fall of 2020.**

**C2. Two embedded questions on the MATH 228 exam 2 in Fall of 2020.**

**(b) Course projects.**
The description of the components of the final project in MATH 200 follow.

*The final paper:* It needs to be about one of two options:

(a) Research proposal: This option would be best for students who have not done any research before or do not have access to a research project readily. The research proposal should include the following sections: introduction, literature review, research question(s), and methods.

(b) Research study: This option would be best for students who are working on a current research project, or have access to a current research project. The research study should include the following sections: introduction, literature review, research question(s), methods, results, and conclusions.

Both projects should use APA citations for in-line citations, and reference lists. The papers should be double spaced, using a reasonable font (i.e. Times/Times New Roman), and should be in the range of 10-15 pages.

*The final presentation:* Students are required to make a 15-30 minute presentation of their final project, utilizing appropriate technology. The recommended software include Power Point, PREZI, or similar software, incorporating visuals, animations, videos, etc. as needed.

Rubrics are included at the end of this report.

Just like in the other direct measures, we will use a universal expectation for all SLOs: every student in the class should score at least 7/10 (70%) on every SLO.

**A3. Final Project for MATH 200 in Fall of 2020.**

**B3. Final Project for MATH 200 in Fall of 2020.**

**B4. Final Project for MATH 200 in Fall of 2020.**

**C3. Final Project for MATH 200 in Fall of 2020.**

3. **Discoveries from data gathered**

<u>*MATH 200:*</u> We assessed one section of this course in the Fall of 2020. Seven students submitted a final project (which includes a presentation). The final project was out of 25

points: 15 for the presentation and 10 points for the written project.
Overall, all students (100%) achieved a 70% in the final project (with scores ranging from 19 to 24). Only one student scored less than 80%, this student ended up with a B in the class; all six others (86% of the class) scored 23 or 24 in the final project, and got As in the class.

The benchmark is: all students get at least a 70% in their project+presentation.

**A3.** **Final written project for MATH 200 in Fall of 2020.** This is part W3 of the written rubric. Please see there what students were expected to achieve.
Following the rubric, three students got a 3, and four got a 4. This means that 7/7 (100%) students met or exceeded expectations.

**B3.** **Final written Project for MATH 200 in Fall of 2020.** This is part W4 of the written rubric. Please see there what students were expected to achieve.
Following the rubric, one student got a 2, five students got a 3, and one got a 4. This means that 6/7 (86%) students met or exceeded expectations.

**B4.** **Final oral and written Project for MATH 200 in Fall of 2020.** These are parts W5 of the written rubric, and P1 and P2 of the presentation rubric. Please see there what students were expected to achieve.
Following the rubric, for W5, one student got a 2, three students got a 3, and three got a 4. This means that 6/7 (86%) students met or exceeded expectations.
For P1, one student got a 2, two students got a 3, and four got a 4. This means that 6/7 (86%) students met or exceeded expectations.
For P2, one student got a 2, one student got a 3, and four got a 4. This means that 6/7 (86%) students met or exceeded expectations.
Overall, the same student got 2s in all three measures. Hence, 6/7 (86%) students met or exceeded expectations.

**C3.** **Final written Project for MATH 200 in Fall of 2020.** These are parts W1 and W2 of the written rubric. Please see there what students were expected to achieve.
Following the rubric, for W1, all students got a 3. This means that 7/7 (100%) students met or exceeded expectations.
For W2, one student got a 2, five students got a 3, and one got a 4. This means that 6/7 (86%) students met or exceeded expectations.
Overall, the student getting a 2 in W2 got a 3 in W1. Hence, 6/7 (86%) students met or exceeded expectations.

**Summary:** In general, students performed well. There was a single student who consistently scored less than the rest. This is the same students who got a B in the class (mentioned at the beginning of this analysis).

*MATH 228:* We assessed one section of this course in the Spring of 2021. Thirteen students

took the midterm and the final exam. Both exams were in-class exams. Each exam was for a total of 100 points, and each question assessed weighted 25 points.
As mentioned before, all questions in these tests were looked at for two different SLOs as follows:
Q1 Exam 2: SLOs C1 and C2.
Q2 Exam 2: SLOs C1 and C2.
Q3 Exam 2: SLOs C1 and B1.
Q1 Final Exam: SLOs A1 and B1.
Q2 Final Exam: SLOs A1 and B1.
Q3 Final Exam: SLOs A1 and C1.

The final grade distribution in this class was: six As, five Bs, and two Cs.

The benchmark is: all students get at least a 70% on each embedded question.

**A1. Three embedded questions on the MATH 228 final exam in Spring of 2021.** Each question being 25 points gives us a maximum of 75 points. Most students scores ranged from 72-75, and only two outliers existed: a 47 and a 65. 70% of 75 is 53; hence 12/13 (92%) students met or exceeded expectations.

**B1. One embedded question on the MATH 228 exam 2 in Fall of 2020.**
**Two embedded questions on the MATH 228 final exam in Spring of 2021.** Each question being 25 points gives us a maximum of 75 points. Then again, 70% of 75 is 53. Although the scores are not as tightly packed as for SLO A1, 12/13 (92%) students met or exceeded expectations. The one student not meeting expectations is the same who did not do so for SLO A1.

**C1. Three embedded questions on the MATH 228 exam 2 in Fall of 2020.**
**One embedded question on the MATH 228 final exam in Spring of 2021.** Each question being 25 points gives us a maximum of 100 points. Although there are four students scoring in the 70s, 12/13 (92%) students met or exceeded expectations. The one student not meeting expectations is **not** the same who did not do so for the previous two SLOs (that student scored a 95 in this SLO).

**C2. Two embedded questions on the MATH 228 exam 2 in Fall of 2020.** Each question being 25 points gives us a maximum of 50 points. Although there are three students scoring 35-38, 12/13 (92%) students met or exceeded expectations. The one student not meeting expectations is the same who did not do so for SLO C1.

**Summary:** There were two students who shared the honors of not meeting expectations for various SLOs. The one who did so for SLOs A1 and B1 (more theoretical ones) passed the class with a B, while the one who did not meet expectations for the C1 and C2 SLO (more applied ones) passed the class only with a C. This seems to indicate that having a good grasp on the applications of the material in this course is important.

*MATH 251:* We assessed one section of this course in the Spring of 2021. Six students took the midterm and the final exam. The midterm was in-class, and it featured five questions for 25 points each. The exam was out of 100, which means that most students would solve four of the five questions in the allotted time. The final was take-home and it had 10 questions for 12 points each, meaning that most students would submit 8-9 questions. Extra credit was possible (25 and 20 points in each of the exams) but the exams were hard and only a few students could earn any extra credit.
Each SLO was measured in each exam to guarantee data, as students did not need to answer all the problems in exams.
Overall, the grade distribution in the class was: three As, two Bs, one C, and three Fs (these three students did not take exam 3 nor the final).

The benchmark is: all students get at least a 70% on each embedded question.

**A1. One embedded question on the MATH 251 exam 3 in Fall of 2020.**
**One embedded question on the MATH 251 final exam in Fall of 2020.** In the Exam 3 question, the scores were 5, 15,15,18,18,18. Since the 70% of 25 is 17.5, we get that 3/6 (50%) of the students met or exceeded expectations.
In the final question, the scores were 7, 7,8,10,12,12. Since the 70% of 12 is 8.4, we get that 3/6 (50%) of the students met or exceeded expectations.
The student who did not meet expectations in Exam 3 did not meet them in the final either.
Overall (adding scores), one gets the scores 12, 25, 25, 26, 27, 30, and thus using that the 70% of 37 is 26, we get that in SLO A1, 3/6 (50%) of the students met or exceeded expectations.

**B1. One embedded question on the MATH 251 exam 3 in Fall of 2020.**
**One embedded question on the MATH 251 final exam in Fall of 2020.** In the Exam 3 question, the scores were 15, 15, 20, 22, 25, 25. Since the 70% of 25 is 17.5, we get that 4/6 (67%) of the students met or exceeded expectations.
In the final question, the scores were 7, 8, 11,12,12,12. Since the 70% of 12 is 8.4, we get that 4/6 (67%) of the students met or exceeded expectations.
Overall (adding scores), one gets the scores 23, 27, 31, 32, 34, 37, and thus using that the 70% of 37 is 26, we get that in SLO A1, 5/6 (83%) of the students met or exceeded expectations.

**C1. One embedded question on the MATH 251 exam 3 in Fall of 2020.**
**One embedded question on the MATH 251 final exam in Fall of 2020.** In the Exam 3 question, the scores were 10, 15, 15,17,25,25. Since the 70% of 25 is 17.5, we get that 2/6 (33%) of the students met or exceeded expectations.
In the final question, the scores were 0, 8,10,10,12, 12. Since the 70% of 12 is 8.4, we get that 4/6 (66%) of the students met or exceeded expectations.
The student who did not meet expectations in Exam 3 did not meet them in the final either.
Overall (adding scores), one gets the scores 17, 22, 23, 25, 35, 37, and thus using

that the 70% of 37 is 26, we get that in SLO A1, 2/6 (33%) of the students met or exceeded expectations.

**Summary:** Individual scores in the embedded questions are not very good. Surprisingly, there were no exam scores, in both Exam 3 and the final, that were below 70%. This seems to be inconsistent with the scores in the embedded questions. The instructor will consider selecting questions for embedding that are at a level similar to the rest of the exam for a next time.

*Exit surveys and interviews:*

In 2020-21, we had our largest graduating class in a while: 12 students. Only seven responded to our survey. As usual, the comments are overall positive. A few suggestions were:

- New, or more variety of graduate courses. Statistics was mentioned, as was something with coding.
- More clear ‘road maps’ for the graduate program.
- Have talks/seminars on how to transition into industry from the M.S. Get internships for grad students.
- Check course prerequisites
- More courses on areas in which faculty are actively active in research.
- More help in selecting a research topic for project/thesis.

At least four students had/will apply to PhD programs in the next year, or so. This is an important increase with respect to previous years.

*Project/Thesis assessment:*

The twelve Projects/Theses delivered this semester were assessed using the rubrics found at the end of this document. Each one of them was assessed by the three members of the corresponding Project/Thesis committee. We use the average of these scores in the table below, and as a point of reference, we mention the score given by the student's advisor.

STUDENT 1: 81%. Score given by advisor: 80%. Project (MATH 298) grade: B.
STUDENT 2: 91%. Score given by advisor: 88%. Project (MATH 298) grade: A.
STUDENT 3: 89%. Score given by advisor: 87%. Project (MATH 298) grade: A.
STUDENT 4: 95%. Score given by advisor: 94%. Project (MATH 298) grade: A.
STUDENT 5: 90%. Score given by advisor: 92%. Project (MATH 298) grade: A.
STUDENT 6: 95%. Score given by advisor: 96%. Thesis (MATH 299) grade: A.
STUDENT 7: 94%. Score given by advisor: 93%. Thesis (MATH 299) grade: A.
STUDENT 8: 84%. Score given by advisor: 82%. Thesis (MATH 299) grade: A.
STUDENT 9: 87%. Score given by advisor: 78%. Thesis (MATH 299) grade: A.

STUDENT 10: 89%. Score given by advisor: 93%. Thesis (MATH 299) grade: A.
STUDENT 11: 94%. Score given by advisor: 96%. Thesis (MATH 299) grade: A.
STUDENT 12: 84%. Score given by advisor: 82%. Thesis (MATH 299) grade: B.

Notice that the average percentage given by the rubrics is not always representative of the final grade awarded to the student. We will explore this situation in the future.

4. **Changes recommended as a result of this year's assessment data**

   Given the special situation under which the 2020-21 AY took place, we do not have many recommendations. Still, a few things are worth mentioning.

   (a) Generate standards for grades given in projects and thesis, and maybe revise our rubric, so the results given by the rubric are consistent with the grade awarded to the student.

   (b) Study the creation of courses that are more on the applied and statistics side of mathematics. Increase the focus our program has in terms of courses that would be useful for somebody wanting to get a job in industry.

   (c) Complete the jobs guide for graduate students.

   (d) Make sure that embedded questions are not only assessing the corresponding SLOs, but also are a good sample of the difficulty the exams where they will be embedded have. In other words, the embedded questions should not always be the hardest ones (or the easiest ones).

5. **Progress made on implementing changes recommended in last year's report**

   There were no recommendations last year because there was no assessment report. Regarding the 2018-19 report, there were two comments/suggestions that were addressed to graduate faculty (more oral presentations and to familiarize students with questions that will appear in exams); these have not been followed up because of the special conditions that we had to teach our courses during the 2020-21 AY. There were two recommendations in the 2018-19 report, they are:

   (a) Create materials that will be used to direct students to career paths. This has not been completed. Some work has been done, though. A guide should be available at the end of this AY.

   (b) Regarding the possibility of running a wider variety of courses, and hopefully more courses. The graduate committee will work this year in designing a plan that will include the phasing out of a couple of courses we currently offer, so a couple of new courses can be created and incorporated into our schedule.

6. **Assessment activities to be conducted in the 2021-22 academic year.**

   (a) Assessment of the SLOs covered by MATH 220, MATH 216T, MATH 260, and MATH 271. This will be done using the terms described in Measure 1 in our SOAP.

(b) We will assess all projects and thesis delivered this academic year using the rubric at the end of this document (Measures 2 and 3).

(c) We will survey our graduating students to learn about their thoughts and feelings about our program (Measure 4).

(d) We will use, for the first time, our new alumni survey (Measure 5).

(e) We will report on Measures 2 and 3, which are the collection of data based on our department rubric for each Project (MATH 298) and Thesis (MATH 299).

(f) We will continue gathering questions that have been used as embedded questions in the past. The idea is that instructors have a source of material they may want to consult at the time they write their own embedded questions.

7. **Major issues identified during the last Program Review, and in what ways they have been addressed**

During the 2016-17 AY, we had a site visit to review our programs. The visit occurred on Sept. 28th and 29th, 2016. The review panel consisted of Prof. Kim Morin, Theatre Arts, CSU Fresno, Dr. Saeed Attar, Professor of Chemistry, Director of Honors College, CSU Fresno, and Dr. Ivona Grzegorczyk, Professor and Chair, Department of Mathematics, California State University, Channel Islands.

The panel delivered several recommendations that have been already discussed in previous year's assessment reports. Next we report **only** on the items for which we reported *no progress* in previous years, and that are relevant to our Master's program.

As expected, the COVID pandemic did not allow any progress in these items.

**B. Supporting Faculty Research and Workload Issues.**

*Recommendation (Administration and the Department).* Identify sources for long term funding so the program can offer release time or summer stipends to faculty engaging in research and grant-writing activities.
*2018-19 response.* No progress.

*This year's response.* No progress.

**C. Departmental Budget.**

*Recommendation.* Identify College and University funds to be included in the departmental funding base for faculty scholarly activities and curriculum coordination.
*2018-19 response.* No progress.

*This year's response.* No progress.

**E. Supporting Undergraduate and Graduate Student Research.**

*Recommendation 2.* Create funding for the department to support small courses for faculty student research projects.
*2018-19 response.* No progress.

*This year's response.* No progress.

*Recommendation 3.* Explore the possibility to offer research courses, where full course load is given to faculty for working with small groups of students.
*2018-19 response.* No progress.

*This year's response.* No progress.

**F. Facilities.**

*Recommendation (Administration) 1.* Try to locate all faculty and graduate student offices in closer proximity to the department.
*2018-19 response.* No progress.

*This year's response.* No progress.

*Recommendation (Administration) 2.* Provide additional space that is equipped appropriately for best practices in teaching mathematics that will facilitate faculty/student collaboration and research activities.
*2018-19 response.* No progress.

*This year's response.* In discussions between the assessment coordinator and the graduate program director, there are several classrooms in Science 2 such as room 308 which facilitate active learning and technology use.

*Recommendation (Administration) 3.* Investigate the use of laptops to meet the computing needs of the faculty and students.
*2018-19 response.* No progress.

*This year's response.* No progress.

**G. Involving Lecturers in Departmental Activities.**

*Recommendation 2.* Allocate an additional appropriate space for the program designated to faculty-student collaborations and projects.
*2018-19 response.* No progress.

*This year's response.* No progress.

**The rubrics for all our assessment activities may be found in the following pages.**

**Rubric for evaluating the written final project**

**and SOAP goals**

**Note: B4 will be mainly assessed at the oral presentation of the project (the use of an appropriate software is a requirement)**

| **Stated objective or criterion** | **Needs Improvement 1** | **Acceptable 2** | **Accomplished 3** | **Exemplary 4** | **SOAP goal** |
|---|---|---|---|---|---|
| **W1 Introduction:** The relevance and the need for the study is well-justified. | Topic is ad-hoc/irrelevant in the field | The study adds some needed new knowledge in the field. | Elements of justification for the relevance of the study and the new knowledge it seeks are present. | The researcher justifies the need for the study and clarifies the new knowledge it adds in the field. | C3 |
| **W2 Research question(s):** Clearly stated, fills a gap in the knowledge base of mathematics education research. | The question(s) is/are vague/the answer(s) is/ are well-known. | The question(s) is/are researchable. The answers have potential interest. | Well-stated question(s). Fills a gap in the knowledge base of mathematics education research. | Well-stated question(s). Fills a significant gap in the knowledge base of mathematics education research. | C3 |
| **W3 Literature review/Theoretical framework:** (a)The merit and relevance of each reviewed work is critically analyzed. (b) Familiarity with the field of study and various theories is evident. | The connections to the proposed study are vague/the cited literature does not fulfill research study norms. | Some connections to the proposed study are justified/the cited literature fulfills research study norms. | Elements of a theoretical framework are included/the cited literature fulfills research study norms. | Provides well-grounded theoretical framework/the cited literature is thorough and fulfills research study norms. | A3 |
| **W4 Research method:** The stated research question(s) is/are answerable by the | The methodology is not able to produce | The methodology is able to produce | The methodology can be carried out based on | The methodology is innovative, described in | B3 |

| | | | | | |
|---|---|---|---|---|---|
| outlined research methodology. | answers to the research question(s). | answers to some aspects of the research question(s). | the detailed description and is able to produce answers to the research question(s). | details, and its appropriateness is justified to produce answers to the research question(s). | |
| **W5 Overall quality of writing:**<br>(a) Follows APA-style;<br>(b) Uses correct grammar, structure, academic language;<br>(c) Length is 10-15 pages;<br>(d) Includes a minimum of 6 reviewed study. | The written final report has major flaws/lacks certain required elements. | The written final report fulfills requirements in (c) and (d), and shows evidence of efforts in satisfying (a) and (b). | The written final report fulfills all requirements at a minimal level. May include some mistakes in (a) and (b). | The written final report fulfills all requirements and exceeds the minimal expectations in (d).<br>May include minor mistakes in (a) and (b). | B4 |

**Rubric for evaluating the presentation of the final project**

**(related to SOAP goal B4)**

| Stated objective or criterion | Needs Improvement 1 | Acceptable 2 | Accomplished 3 | Exemplary 4 | Score |
|---|---|---|---|---|---|
| **P1 Use of software:** The chosen software is appropriate to support the flow of the presentation. | The use of the software does not enhance the quality of the presentation (same as writing on the board) | The presentation includes some visual (or other) enhancement compared to just writing on the board. | The presentation is supported by charts/graphs or other visuals or other enhancements to support the presentation. | The presentation looks professional, includes informative visuals or other enhancements, the power of the technology is well-utilized. | |
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| | | | | | |
| | | | | | |
| **P2 Overall quality of the presentation:** (a) Uses correct grammar, academic language; (b) Length is 15-30 minutes; (c) The flow, content, and pace of the presentation is appropriate to the needs of the audience; (d) The presenter effectively uses technology to produce a high quality presentation | The presentation has major flaws/lacks certain required elements. | The presentation fulfills two of the requirements at a minimal level and shows evidence of efforts in satisfying the others. | The presentation fulfills all requirements at a minimal level. May include minor mistakes in some areas. | The presentation fulfills all requirements and exceeds the minimal expectations in at least one area. May include minor mistakes. | |

## PROBLEMS FOR SOAP

**General information:** Exam 2 and the Final Exam are in class closed note exams and they contain 4 problems each, with 25 points for each problem. Exam 2 contains problems 3, 4, 5, and one more problem. Final exam will contain problems 1, 2, 6, and one more problem. In both exams, three of the four problems come from (or with slight modifications) from homework problems. Although these three problems are not easy, I expect most students will be able to solve these three problems since they have worked on them before as homework problems with feedback on their solutions and my solutions to homework problems have been posted on Canvas. With these three problems, I expect students will have at least 75 points to have at least a C in the course. There is one more problem that students have not seen before or this problem may be a portion of a long big proof in the lecture. I expect that a "B" student will be able to have an initial set up of the problem to bring their score to between 80-90. An "A" student should be able to make a big progress or solve this whole problem to bring the exam grade to 90-100. I put the details of which problems students have done in homework below.

**Problem 1** (A1, B1. This is a portion of the long proof of the famous Open Mapping Theorem in the lecture. Students have not worked on this as homework. I tell the students where to start to finished the theorem.)**.** Background: Suppose $f(z)$ is non-constant analytic on $B_R(a)$, and $f(z)-\alpha$ has a zero at $z=a$ of multiplicity $m$. There exist $\epsilon > 0$ and $\delta > 0$ so that if $\zeta \in B(a,\delta)\backslash\{a\}$, then the function $f(z)-\zeta$ has exactly $m$ simple zeros in $B(a,\epsilon)$.
Use the background above to prove the Open Mapping Theorem which says if $f(z)$ be a non-constant analytic function in an open connected domain $G$, then for any open set $U \subseteq G$, the set $f(U)$ is open.

*Proof.* □

(1) (5 points) Let $\alpha \in f(U)$ be arbitrary. We will show that there is $\delta > 0$ so that $B_\delta(\alpha) \subseteq f(U)$.
(2) (5 points) Since $\alpha \in f(U)$, there is $a \in U$ such that $f(a) = \alpha$. From the assumption that $U$ is open, there is $R > 0$ so that $B_R(a) \subseteq U$.
(3) (5 points) Since $f(a) = \alpha$, the function $f(z) - \alpha$ has a zero $z = a$ of multiplicity $m \geq 1$. By the background applied to $f(z)$ on $B_R(a)$, there exist $\epsilon > 0$ and $\delta > 0$ so that if $\zeta \in B(\alpha,\delta)\backslash\{\alpha\}$, the function $f(z)-\zeta$ has exactly $m$ simple zeros in $B(a,\epsilon)$.
(4) (5 points) With the $\epsilon$ and $\delta$ above, the inequality $m \geq 1$ and $f(a) = \alpha$ imply that for each $\zeta \in B_\delta(\alpha)$, the equation $f(z)-\zeta$ has at least one solution in $B(a,\epsilon)$.
(5) (5 points) Thus $B_\delta(\alpha) \subseteq f(B_\epsilon(a)) \subseteq f(U)$ and consequently $f(U)$ is an open set.

**Problem 2** (A1,B1. This is a rather hard problem but students have worked on this as homework)**.** In this problem, we will prove the generalized Schwarz's Lemma. Suppose $f(z)$ is analytic on $B_1(0)$ such that

- $|f(z)| \leq 1 \ \forall z \in B_1(0)$ and
- $f(z)$ has a zero $z = 0$ of multiplicity $n$.

The goal is to prove that $|f(z)| \leq |z|^n \ \forall z \in B(0,1)$ and $|f^{(n)}(0)| \leq n!$. And moreover, if $|f(z_0)| = |z_0|^n$ for some $z_0 \in B_1(0)\backslash\{0\}$ or $|f^{(n)}(0)| = n!$ then $f(z) \equiv cz^n$ for some $|c| = 1$. Follow the steps below.

(1) Let

$$g(z) = \begin{cases} f(z)/z^n & \text{if } z \neq 0 \\ \frac{f^{(n)}(0)}{n!} & \text{if } z = 0 \end{cases}.$$

Prove that $g(z)$ is analytic on $B(0,1)$ by showing that $g(z)$ is continuous at 0.

(2) Prove that $|g(z_0)| \leq 1$ for any $z_0 \in B(0,1)$ and deduce that $|f(z_0)| \leq |z_0|^n$ and and $|f^{(n)}(0)| \leq n!$.

(3) Prove that if $|f(z_0)| = |z_0|^n$ for some $z_0 \in B_1(0)\backslash\{0\}$ or $|f^{(n)}(0)| = n!$ then $f(z) = cz^n$ $\forall z \in B_1(0)$ for some $|c| = 1$.

*Proof.*

(1) (10 points) Since $z = 0$ is a zero of multiplicity $n$, $f(z)$ has the power series

$$f(z) = z^n(a_0 + a_1 z + \dots)$$

for $z \in B_1(0)$. From the formula of the coefficient of the power series, we note that

$$a_0 = \frac{f^{(n)}(0)}{n!} \neq 0.$$

Since

$$\lim_{z\to 0} \frac{f(z)}{z^n} = \lim_{z\to 0} (a_0 + a_1 z + \dots) = a_0 = \frac{f^{(n)}(0)}{n!},$$

$g(z)$ is continuous at 0 and thus $g(z)$ is analytic on $B(0,1)$.

(2) (10 points) For any $z_0 \in B(0,1)$, we choose $r$ where $z_0 \in B(0,r)$. By the Maximum Modulus principle, we have

$$|g(z_0)| \leq \max_{|z|=r} |g(z)| = \max_{|z|=r} \frac{|f(z)|}{|z|^n} \leq \frac{1}{r^n}.$$

We let $r \to 1^-$ and have $|g(z_0)| \leq 1$. From the definition of $g(z)$, the inequality, this inequality implies $|f(z_0)| \leq |z_0|^n$ and $|f^{(n)}(0)| \leq n!$.

(3) (5 points) If $|f(z_0)| = |z_0|^n$ for some $z_0 \in B(0,1)\backslash\{0\}$ or $|f^{(n)}(0)| = n!$ then we have $z_0 \in B(0,1)$ where $1 = |g(z_0)| \geq |g(z)| \ \forall z \in B_1(0)$. By the Maximum Modulus Principle applied to $g(z)$ on $B_1(0)$, $g(z)$ is the constant function $g(z) \equiv c$ for some $|c| = 1$. This implies $f(z) = cz^n \ \forall z \in B_1(0)$.

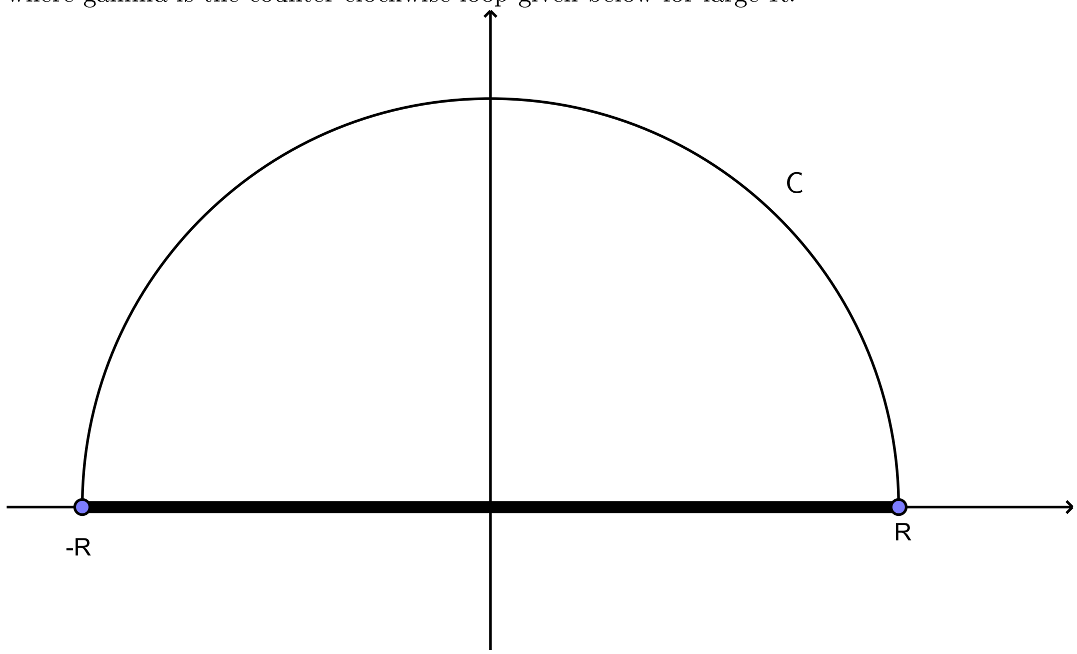
□

**Problem 3** (C1, C2. This is a slight modification of a homework problem)**.** The goal of this problem is to apply complex integral to find the area between the curve $x^2/(x^4+1)$ and the real axis. Follow the steps below.

(1) Evaluate

$$\oint_\gamma \frac{z^2 dz}{z^4+1}$$

where gamma is the counter clockwise loop given below for large $R$.




(2) Let $C$ be the semicircle (radius $R$) arc on the upper half plane with counter clockwise orientation. Show that

$$\lim_{R\to\infty}\int_C \frac{z^2dz}{z^4+1} = 0.$$

(3) Explain how the results in part (1) and (2) help us find the area between the curve $x^2/(x^4+1)$ and the real axis.

*Proof.*

(1) (10 points) We can deform $\gamma$ into two loops $\gamma_1$ around $e^{i\pi/4}$ and $\gamma_2$ around $e^{3i\pi/4}$ with radius 1/10. Thus

$$\begin{aligned}\oint_\gamma \frac{z^2dz}{z^4+1} &= \oint_{\gamma_1}\frac{z^2dz}{z^4+1} + \oint_{\gamma_2}\frac{z^2dz}{z^4+1}\\ &= 2\pi i\frac{(e^{i\pi/4})^2}{4e^{3i\pi/4}} + 2\pi i\frac{(e^{3i\pi/4})^2}{4e^{9i\pi/4}}\\ &= \frac{2\pi i}{4}.(e^{-i\pi/4}+e^{-3i\pi/4})\\ &= \frac{2\pi i}{4}.(-\sqrt{2}i) = \frac{\pi}{\sqrt{2}}.\end{aligned}$$

where in the second step we apply Cauchy integral formula by computing the derivative of $z^4+1$ and evaluate and $e^{i\pi/4}$ and $e^{3i\pi/4}$.

(2) (10 points) We note that

$$\left|\int_C \frac{z^2dz}{z^4+1}\right| \le \int_C \frac{|z|^2|dz|}{|z^4+1|} \le \int_C \frac{|z|^2|dz|}{||z|^4-1|} = \int_0^\pi \frac{R^2.Rdt}{R^4-1} = \frac{\pi R^3}{R^4-1} \to 0$$

as $R \to \infty$ by the L'Hopital Rule.

(3) (5 points) The area in this problem is

$$\int_{-\infty}^{\infty} \frac{dx}{x^4+1} = \lim_{R\to\infty} \int_{-R}^{R} \frac{dx}{x^4+1} = \lim_{R\to\infty} \left( \int_{\gamma} \frac{dz}{z^4+1} - \int_{C} \frac{dz}{z^4+1} \right) = \frac{\pi}{\sqrt{2}}$$

since by part (1)

$$\int_{\gamma} \frac{dz}{z^4+1} = \frac{\pi}{\sqrt{2}}$$

and by part (2)

$$\lim_{R\to\infty} \int_{C} \frac{dz}{z^4+1} = 0.$$

□

**Problem 4** (C1,C2. This is a homework problem)**.** The goal of this problem is to apply the Identity Theorem to demonstrate that all trigonometric identities we learned for real variables will work for complex variables as well. Assume that $\cos(a+b) = \cos a \cos b - \sin a \sin b$ is valid for all $a, b \in \mathbb{R}$. Use the Identity Theorem to prove that this formula holds for $a, b \in \mathbb{C}$. Do NOT splitting real, imaginary parts of $a, b$ or definition of sine and cosine function as we want to have a method that works for all trigonometric identities, not just this one.

*Proof.* □

(1) (5 points) For any $\underline{\text{fixed } b \in \mathbb{R}}$, we consider the entire function $f(z) = \cos(z+b) - (\cos z \cos b - \sin z \sin b)$.
(2) (5 points) Since the set of zeros of $f(z)$ contains $\mathbb{R}$ which has a limit point, we deduce from the Identity Theorem that $f(z) = 0\ \forall z \in \mathbb{C}$.
(3) (5 points) Consequently,

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

is valid for all $\underline{a \in \mathbb{C} \text{ and } b \in \mathbb{R}}$.
(4) (5 points) Now for any fixed $a \in \mathbb{C}$, we consider the entire function $g(z) = \cos(a+z) - (\cos a \cos z - \sin a \sin z)$.
(5) (5 points) The set of zeros of $g(z)$ contains $\mathbb{R}$ by the previous part, we will have $g(z) = 0$ for all $z \in \mathbb{C}$. Thus

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

for all $a, b \in \mathbb{C}$.

**Problem 5** (C1,B1. This is a new problem; students have not worked on this problem before. I expect a "B" student will be able to do part 1)**.** Let $G$ be an open connected domain and $f, g : G \to \mathbb{C}$.

(1) Give an example of an open connected domain $G$ and two functions $f, g$ (do NOT have to be analytic or continuous on $G$) where $f(z)g(z) = 0\ \forall z \in G$ but neither $f$ nor $g$ are constant 0 function on $G$.
(2) Apply Identity Theorem to prove in that if $f$ and $g$ (not the functions in part (1)) are analytic on $G$ and $f(z)g(z) = 0$ for all $z \in G$ then either $f$ or $g$ is the constant zero function on $G$.

*Proof.*

(1) (10 points) We consider an example where $G = \mathbb{C}$, and

$$f(z) = \begin{cases} 1 & \text{if } z = 0 \\ 0 & \text{if } z \neq 0 \end{cases},$$

$$g(z) = \begin{cases} 0 & \text{if } z \neq 0 \\ 1 & \text{if } z = 0 \end{cases}.$$

Clearly $f(z)g(z) = 0\ \forall z \in \mathbb{C}$ but neither $f$ nor $g$ are constant 0 function.

(2)

- (5 points) Let $a \in G$. Since $G$ is open there is $B_r(a) \subseteq G$ for some $r > 0$. Since $f(z)g(z) = 0\ \forall z \in B_r(a)$, we have

$$\{z \in B_r(a)|f(z) = 0\} \cup \{z \in B_r(a)|g(z) = 0\} = B_r(a).$$

- (5 points) Since $B_r(a)$ is an infinite set either $\{z \in B_r(a)|f(z) = 0\}$ or $\{z \in B_r(a)|g(z) = 0\}$ is an infinite set. WLOG, suppose $\{z \in B_r(a)|f(z) = 0\}$ is an infinite set.
- (5 points) Since this set is bounded, by the Bolzano-Weierstrass theorem, it has a limit point. By the identity theorem applied to $f$ on the whole domain $G$, $f \equiv 0$ on $G$.

□

**Problem 6** (A1,C1. This is a homework problem)**.** Background: Let $G$ be a region (connected and open) and $g_n(z) : G \to \mathbb{C}$ be analytic. If $\sum_{n=1}^{\infty} g_n(z)$ converges absolutely and uniformly on any compact subset of $G$ then $\prod(1 + g_n(z))$ converges absolutely to an analytic function in $G$ and the convergence is uniform on any compact subset of $G$.

Use the background above for the problem below.

(1) Let $0 < |a| < 1$ and $|z| \leq r < 1$. Show that

$$\left| \frac{a + |a|z}{(1 - \overline{a}z)a} \right| \leq \frac{1 + r}{1 - r}.$$

(2) Let $(a_n)$ be a sequence of complex numbers with $0 < |a_n| < 1$ and $\sum(1 - |a_n|)$ converges. Show that the product

$$\prod_{n=1}^{\infty} \frac{|a_n|}{a_n} \left( \frac{a_n - z}{1 - \overline{a_n}z} \right)$$

converges absolutely to an analytic function on $B_1(0)$ and the convergence is uniform on any compact subset of $B_1(0)$.

*Proof.*

(1) (10 points) By the triangle inequality, we have

$$\left| \frac{a + |a|z}{(1 - \overline{a}z)a} \right| \leq \frac{|a| + |a||z|}{(|1 - |\overline{a}||z|)|a|} = \frac{1 + |z|}{1 - |\overline{a}||z|} \leq \frac{1 + r}{1 - r}.$$

(2)

- (5 points) Let $K$ be any compact subset of $B(0,1)$. Since $K$ is compact, there is $r<1$ such that $K \subset B(0,r)$. By the background theorem, it now suffices to prove the absolute and uniform convergence on $K$ of
$$\sum \left( \frac{|a_n|}{a_n} \left( \frac{a_n - z}{1-\overline{a_n}z} \right) - 1 \right).$$
- (5 points) The modulus of the summand is
$$\begin{aligned} \left| \frac{|a_n|(a_n - z) - a_n(1-\overline{a_n}z)}{a_n(1-\overline{a_n}z)} \right| &= \left| \frac{|(|a_n|-1)(a_n + |a_n|z)}{a_n(1-\overline{a_n}z)} \right| \\ &\leq (1-|a_n|)\frac{1+r}{1-r} \end{aligned}$$
where the last inequality comes from the previous part.
- (5 points) Since the series of numbers
$$\frac{1+r}{1-r}\sum(1-|a_n|)$$
converges by hypothesis, the series
$$\sum \left( \frac{|a_n|}{a_n} \left( \frac{a_n - z}{1-\overline{a_n}z} \right) - 1 \right)$$
converges uniformly on $K$ by the Weierstrass $M$-test. Also by the comparison test, this series converges absolutely on $K$. The claim follows.

$\square$

## MATH 251, Exam 3 and Final Assessment Problems

MATH 251 needs to assess the following SLOs.

**Student Learning Outcomes.** Graduates will be able to:

- **A1.** learn and understand advanced concepts in abstract algebra, analysis, and other pure mathematics topics.
- **B1.** reconstruct proofs of classical theorems and/or write rigorous, multi-concept proofs using advanced concepts in pure mathematics, such as transformation, Lebesgue integral, uniform convergence, matrix groups, topological spaces, algebraic structures, etc.
- **C1.** apply the structural relationships among the various advanced concepts in pure mathematics to solve problems.

**Problems for Exam 3 (by SLO).**

This exam is in-class only. Each question will be 20 points.

1. **SLO A1.** Let $R$ be a PID and $I$ an ideal of $R$.
   **(a)** Prove that the every ideal in the quotient $R/I$ is principal.
   **(b)** Given an example in which $R/I$ is not a PID.

   **Rubric:**
   - State that ideals of $R/I$ are in correspondence with the ideals of $R$ that contain $I$, via the standard projection $\pi : R \to R/I$. **5 points.**
   - Consider an ideal $J$ in $R/I$, use part (a) to get $K$, ideal of $R$ so that $K/I = J$. **5 points.**
   - Since $K$ is principal then so is $J$, as $\pi$ maps generators to generators and $\pi$ is onto. **5 points.**
   - $\mathbb{Z}/n\mathbb{Z}$ is an example, for $n$ composite, as it contains zero divisors. **5 points.**

2. **SLO B1.** Consider the ring $\mathcal{F} = \{f : \mathbb{R} \to \mathbb{R};\ f \text{ is a function}\}$, with operations given by

   $$(f+g)(x) = f(x) + g(x) \quad \forall x \in \mathbb{R} \qquad\qquad (fg)(x) = f(x)g(x) \quad \forall x \in \mathbb{R}$$

   for all $f, g \in \mathcal{F}$.
   **(a)** Prove that every non-zero function in $\mathcal{F}$ is either a unit or a zero divisor.
   **(b)** Prove that $J = \{f \in \mathcal{F};\ f \text{ is continuous}\}$ is a subring of $\mathcal{F}$ but it is not an ideal of $\mathcal{F}$.

**Rubric:** Part (a) was partially discussed/proved in class as an example.

- Explicitly describe what 0 and 1 are in $\mathcal{F}$, and then conclude that units are functions that are never zero. **5 points.**
- For a function that is zero at some $a \in \mathbb{R}$, construct a function that is zero everywhere else, but not at $a$. This shows that every non-zero function that is zero at at least one point must be a zero divisor. **5 points.**
- Use subring test, or prove directly that $J$ is a subring of $\mathcal{F}$ (not hard, just need to know what to do). **5 points.**
- Provide example of a discontinuous function times a continuous function yielding a discontinuous function. **5 points.**

3. **SLO C1.** Let $R$ be an integral domain. Prove that there are at most two ring homomorphisms $\phi : S \to R$, where $S$ is either $\mathbb{Z}$ or $\mathbb{Z}_n$ (for some $n$).

**Rubric:**

- State that $\phi$ is uniquely determined by $\phi(1)$. Thus, $\phi(x) = nx$, where the operation on the right-hand side is done using the operation in $R$. **5 points.**
- Prove that $\phi(x) = nx$ is always a group homomorphism. **5 points.**
- Check that imposing
$$\phi(xy) = \phi(x)\phi(y)$$
forces $n^2 = n$ (using that $R$ is commutative and that it has no zero divisors). Since $R$ is an integral domain $n = 0$ or $n = 1$. Hence, $\phi = Id$ or $\phi \equiv 0$. **10 points.**

**Problems for Final Exam.**

This exam is take-home only. Each question will be 12 points.

1. **SLO A1.** A group $G$ is said to be metabelian if it has a normal Abelian subgroup $N$ such that $G/N$ is Abelian.
**(a)** Show that $S_3$ is metabelian.
**(b)** Show that every subgroup of a metabelian group is metabelian.
**(c)** Show that any homomorphic image of a metabelian group is metabelian.

**Rubric:**

- Part **(a)**. Just take $N = A_3$. Clearly $N \cong \mathbb{Z}_3$ and thus abelian. $A_n$ is always normal in $S_n$ and the quotient has order 2, and thus isomorphic to $\mathbb{Z}_2$. **2 points.**
- Part **(b)**. If $N$ is a normal subgroup of $G$ so that both $N$ and $G/N$ are abelian, and $H \leq G$ then $N \cap H$ is abelian and normal in $H$. Using the second isomorphism

theorem, we get that $HN/N \cong H/H \cap N$. Since $HN/N$ is a subgroup of $G/N$, it is abelian. The result follows. **5 points.**

- Part **(c)**. Assume that $N$ is a normal subgroup of $G$ so that it and $G/N$ are abelian. Consider $H = \phi(G)$ and $K = \phi(N)$. By the correspondence, AKA fourth isomorphism theorem, (using $H$ and $\{1\}$) we get that $K$ is normal in $H$. Moreover, for $aK, bK \in H/K$, we let $a = \phi(g)$ and $b = \phi(h)$, for some $g, h \in G$. Since $G/N$ is abelian, we get that

$$(gh)N = (gN)(hN) = (hN)(gN) = (hg)N$$

and thus $h^{-1}g^{-1}hg \in N$.Then,

$$b^{-1}a^{-1}ba = \phi(h)^{-1}\phi(g)^{-1}\phi(h)\phi(g) = \phi(h^{-1}g^{-1}hg) \in \phi(N) = K$$

and thus

$$(aK)(bK) = (ab)K = (ba)K = (bK)(aK)$$

which implies that $H/K$ is abelian. **5 points.**

2. **SLO B1.** Assume that $K$ is a normal subgroup of the finite group $G$ and that $P \in Syl_p(K)$. Prove that $G = N_G(P)K$.

**Rubric:** This is the famous Frattini argument. I have used this problem twice for assessment in the past. It might be good to see it again.

- **2 point**. We note first that $K \cdot N_G(P) \subset G$ (since $K \subset G$, $N_G(P) \subset G$ and $G$ is closed under composition). Let $g \in G$. Since $P \subset K$ and $K$ is normal in $G$, then $gPg^{-1} \subset K$. Moreover, $gPg^{-1} \leq K$ and $|gPg^{-1}| = |P|$. Thus, $gPg^{-1} \in Syl_p(K)$.
- **3 points**. Then, by Sylow's Theorem for the group $K$, there exists $n \in K$ such that $gPg^{-1} = nPn^{-1}$.
- **3 points**. Then, $n^{-1}gPg^{-1}n = n^{-1}nPn^{-1}n = P \Longrightarrow (n^{-1}g)P(n^{-1}g)^{-1} = P$.
- **4 points**. It follows that $n^{-1}g \in N_G(P)$, which implies that $g \in nN_G(P)$. Since $g$ was arbitrarily chosen, we have shown that $G \subset K \cdot N_G(P)$, and therefore, $G = K \cdot N_G(P)$, which using that $|K \cdot N_G(P)| = |N_G(P)K|$ implies that $G = N_G(P)K$.

3. **SLO C1.** Let $|G| = (pq)^2$, where $p$ and $q$ are primes such that $q = p + 2$, and $p > 3$. Prove that $G$ is Abelian. Classify these groups.

**Rubric:** Very typical application of the Sylow Theorems and the classification of finite abelian groups.

- Using Sylow, we get that $n_q = 1$ ($p^2 \equiv 4 \pmod q$, and $q \geq 5$), and so $P_q$ is normal in $G$. **3.5 points.**
- Since $p > 3$ then $n_p = 1$ as well ($q^2 \equiv 4 \pmod p$). **3.5 points.**
- In this case, it follows that $G = P_p \times P_q$. **2 points.**

- It is known that groups of order $p^2$, where $p$ is prime must be isomorphic to $\mathbb{Z}_p \times \mathbb{Z}_p$ or $\mathbb{Z}_{p^2}$. Hence, all the possibilities for $G$ are $G = \mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_q \times \mathbb{Z}_q$, $\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_{q^2}$, $\mathbb{Z}_{p^2} \times \mathbb{Z}_{q^2}$ and $\mathbb{Z}_{p^2} \times \mathbb{Z}_q \times \mathbb{Z}_q$. Of course, all of them abelian. **3 points.**

**Thesis Evaluation Rubric**

**Student's name: ____________________________________________**

**Assessed by: __________________________________________ Date: ______________**

| **Attribute** | **Inadequate (1)** | **Adequate (2)** | **Good (3)** | **Excellent (4)** |
|---|---|---|---|---|
| **Quality of Mathematics**<br><br>**Score:**<br><br>**___/24** | ☐ Inadequate literature review, lacking mathematical context | ☐ Literature review is present, but incomplete mathematical context for thesis given | ☐ The student has reviewed the literature and explained how thesis fills a gap | ☐ The student has reviewed the literature and explained how thesis fills a gap and extends the field |
| | ☐ No potential for publication | ☐ Much work required for potential publication | ☐ Some work required for potential publication | ☐ Little work required for potential publication |
| | ☐ Purpose of the work is unclearly or inadequately explained | ☐ Purpose of the work is adequately explained, but may be disorganized | ☐ Purpose of the work is clearly explained | ☐ Purpose of the work is clearly and thoroughly explained |
| | ☐ Mathematical analysis is inadequate or inadequately explained | ☐ Mathematical analysis is adequately explained | ☐ Mathematical analysis is mostly comprehensive | ☐ Mathematical analysis is thorough and extensive |
| | ☐ No attempt to place results into broader context | ☐ Placing of results into broader context is included, but not comprehensive | ☐ Implications and placing results into a broader context are comprehensively discussed | ☐ Implications and placing results into a broader context are comprehensively and insightfully discussed |
| | ☐ No attempt to discuss future directions if applicable | ☐ Applicable future directions discussed, but not comprehensively | ☐ Applicable future directions are comprehensively discussed | ☐ Applicable future directions are comprehensively and insightfully discussed |

| Attribute | Inadequate (1) | Adequate (2) | Good (3) | Excellent (4) |
|---|---|---|---|---|
| **Quality of Writing**<br><br>**Score:**<br><br>**___/20** | ☐ Writing is not understandable and/or engaging to readers<br><br>☐ Writing is not organized, inconsistent and does not flow in a logical manner<br><br>☐ Thesis contains excessive spelling/grammar errors<br><br>☐ Organization of thesis does not follow standard mathematical format<br><br>☐ The writing within one section often belongs in another section | ☐ Writing could be clearer and more engaging.<br><br>☐ It is possible, but not easy, to follow the main themes of the thesis as the writing is mostly logical and consistent<br><br>☐ Thesis is somewhat free from spelling and grammar errors<br><br>☐ Organization of paper somewhat follows standard mathematical format<br><br>☐ The writing within each section is generally appropriate for that section | ☐ Writing is understandable and engaging to the reader<br><br>☐ Writing is organized, consistent, and logical with main themes that are easy to follow.<br><br>☐ Thesis is virtually free from spelling and grammar errors<br><br>☐ Organization of thesis mostly follows standard mathematical format<br><br>☐ The writing within each section is mostly appropriate for that section | ☐ Writing is exceptional, clearly understandable, and engaging to the reader<br><br>☐ Writing is exceptionally well organized, consistent, and logical with main themes that are easy to follow.<br><br>☐ Thesis is free from spelling and grammar errors<br><br>☐ Organization of thesis follows standard mathematics format<br><br>☐ The writing within each section is always appropriate for that section |

| Attribute | Inadequate (1) | Adequate (2) | Good (3) | Excellent (4) |
| --- | --- | --- | --- | --- |
| **Quality of Presentation**<br><br>**Score:**<br><br>**____/12** | ☐ Slides are unclear, incorrect, or misleading<br><br>☐ Oral presentation is incomplete or unclear<br><br>☐ Citations are missing or inconsistent | ☐ Slides are somewhat clear and appropriate<br><br>☐ Oral presentation is somewhat clear and complete<br><br>☐ Others' work is referenced consistently, although a few errors exist | ☐ Slides are clear and appropriate<br><br>☐ Oral presentation is mostly clear and complete<br><br>☐ Citations of others' work are mostly consistent and appropriate | ☐ Slides are clear and help improve attendees' understanding<br><br>☐ Oral presentation is very clear and complete<br><br>☐ Citations for others' work are consistent and appropriate |
| **Overall Assessment** | ☐ Inadequate | ☐ Adequate | ☐ Good | ☐ Excellent |

**TOTAL: ___/56** **Percentage: ___**

**Additional Notes:**

## Project Evaluation Rubric

**Student's name:** ______________________________

**Assessed by:** ______________________________ **Date:** ______________

| Attribute | Inadequate (1) | Adequate (2) | Good (3) | Excellent (4) |
|---|---|---|---|---|
| **Quality of Mathematics**<br><br>**Score:**<br><br>**___/20** | ☐ Inadequate literature review, lacking mathematical context | ☐ Literature review is present, but mathematical context for current work is not complete | ☐ The student has reviewed the literature and explained how current work fits into this context | ☐ The student has reviewed the literature and explained how current work fills a gap |
| | ☐ Purpose of the work is unclearly or inadequately explained | ☐ Purpose of the work is adequately explained, but may be disorganized | ☐ Purpose of the work is clearly explained | ☐ Purpose of the work is clearly and thoroughly explained |
| | ☐ Mathematical analysis is inadequate or inadequately explained | ☐ Mathematical analysis is adequately explained | ☐ Mathematical analysis is mostly comprehensive | ☐ Mathematical analysis is thorough and extensive |
| | ☐ No attempt to place results into broader context | ☐ Placing of results into broader context is included, but not comprehensive | ☐ Placing results into broader context is comprehensively discussed | ☐ Placing results into broader context is comprehensively and insightfully discussed |
| | ☐ No attempt to discuss future directions if applicable | ☐ Applicable future directions are discussed, but are not comprehensive | ☐ Applicable future directions are comprehensively discussed | ☐ Applicable future directions are comprehensively and insightfully discussed |

| Attribute | Inadequate (1) | Adequate (2) | Good (3) | Excellent (4) |
| --- | --- | --- | --- | --- |
| **Quality of Writing**<br><br>**Score:**<br><br>**___/20** | ☐ Writing is not understandable and engaging to readers<br><br>☐ Writing is not organized, inconsistent and does not flow in a logical manner<br><br>☐ Paper contains excessive spelling/grammar errors<br><br>☐ Organization of paper does not follow standard mathematical format<br><br>☐ The writing within each section often belongs in another section | ☐ Writing can be clearer and more engaging.<br><br>☐ It is possible, but not easy, to follow the main themes of the paper as the writing is mostly logical and consistent<br><br>☐ Paper is somewhat free from spelling and grammar errors<br><br>☐ Organization of paper somewhat follows standard mathematical format<br><br>☐ The writing within each section is generally appropriate for that section | ☐ Writing is understandable and engaging to the reader<br><br>☐ Writing is organized, consistent, and logical with main themes that are easy to follow.<br><br>☐ Paper is virtually free from spelling and grammar errors<br><br>☐ Organization of paper mostly follows standard mathematical format<br><br>☐ The writing within each section is mostly appropriate for that section | ☐ Writing is outstanding, clearly understandable, and engaging to the reader<br><br>☐ Writing is extremely well organized, consistent, and logical with main themes that are easy to follow.<br><br>☐ Paper is free from spelling and grammar errors<br><br>☐ Organization of paper follows standard mathematics format<br><br>☐ The writing within each section is appropriate for that section |

| Attribute | Inadequate (1) | Adequate (2) | Good (3) | Excellent (4) |
|---|---|---|---|---|
| **Quality of Presentation**<br><br>**Score:**<br><br>**____/12** | ☐ Slides are unclear, incorrect, or misleading<br><br>☐ Oral presentation is incomplete or unclear<br><br>☐ Citations are missing or inconsistent | ☐ Slides are somewhat clear and appropriate<br><br>☐ Oral presentation is somewhat clear and complete<br><br>☐ Other's work is referenced consistently, although a few errors exist | ☐ Slides are clear and appropriate<br><br>☐ Oral presentation is mostly clear and complete<br><br>☐ Citations of others' work are mostly consistent and appropriate | ☐ Slides are clear and help improve attendees' understanding<br><br>☐ Oral presentation is very clear and complete<br><br>☐ Citations for others' work are consistent and appropriate |
| **Overall Assessment** | ☐ Inadequate | ☐ Adequate | ☐ Good | ☐ Excellent |

**TOTAL: ____/52** **Percentage: ___**

**Additional Notes:**