What is a Hadamard Matrix (and Why Should You Care)?

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In 1893 Hadamard asked the following: If H is an n by n real matrix whose entries h_{ij} have absolute value ≤ 1, what is the maximum absolute value of the determinant of H? Hadamard proved that $|\det H| \leq n^{n/2}$ with equality iff (a) $h_{ij} = \pm 1$, and (b) any two distinct rows (columns) of H are mutually orthogonal. A matrix H satisfying conditions (a) and (b) is called a *Hadamard* matrix. [The preceding is a linear algebraic definition, but Hadamard matrices also have a beautiful combinatorial interpretation: if you take any two distinct rows (columns) and you place one on top of the other, there are an equal number of "matches" and "mismatches."]

It is not hard to prove that the only possible orders for a Hadamard matrix are $n = 2$ or $n = 4k$. The converse of this fact is the mother of all Hadamard matrix questions: Are there Hadamard matrices of order n for all $n = 4k$? It is generally believed that the answer is Yes, but progress in proving this 100+ year old conjecture has been very slow. In this talk I will give a general overview on what is known (and mostly not known) about the existence of Hadamard matrices.