

Solution to Problem
of the three weeks
December '09

α_1, α_2 are roots of
 $ax^2 + bx + c = 0$

β_1, β_2 are roots of
 $px^2 + qx + r = 0$

$\left. \begin{array}{l} \alpha_1 y + \alpha_2 z = 0 \\ \beta_1 y + \beta_2 z = 0 \end{array} \right\}$ has a non-trivial solution

$$\Rightarrow \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0$$

ie. $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$

$$\Rightarrow \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \frac{\sqrt{\alpha_1 \alpha_2}}{\sqrt{\beta_1 \beta_2}}$$

$$\Rightarrow \frac{-b/a}{-q/p} = \frac{\sqrt{c/a}}{\sqrt{r/p}} \Rightarrow \boxed{\frac{b^2}{q^2} = \frac{ac}{pr}} \quad \blacksquare$$