Math Field Day 2024

Mad Hatter Marathon 9-10

CSU Fresno Mathematics
www.fresnostate.edu/csm/math/

April 20, 2024

Mad Hatter Marathon 9-10

Welcome to Fresno State!

The Mad Hatter Marathon is a competition in rapid computation and problem solving. You may find that you do not have time to solve every problem. After a few minutes you may feel "mentally out of breath." Do not let this discourage you. Your fellow contestants feel the same way. That is why this contest is called *Mad Hatter Marathon!*

Mad Hatter 9-10

The Mad Hatter Marathon is divided into two problem solving periods, each lasting 60 minutes. Between the two periods there will be a 15-minute break.

Part I

- This part of the competition consists of 30 problems.
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Rules and Scoring

You may use pencil and scratch paper to do calculations, but **calculators are not allowed.**

Your score is the total number of correct answers, so give the best answer that you can in the time available for each problem. There is no penalty for guessing.

Filling Out Your Scantron

- Put your **full name** in the box labeled “Name.”
- Put your **full school name** in the box labeled “Subject.”
- Write “**Part 1**” in the box labeled “Exam.”

Reminders

- Please turn off any devices that could make noise, such as cell phones, beepers, watches, etc.
- If your pencil breaks or needs sharpening, stay in your seat and raise your hand.
- Keep your eyes on your own paper. Keep your Scantron flat on your desk. Contestants caught cheating will be disqualified.

Ready... Set... Go!

Prepare to begin the *Mad Hatter Marathon!*

Part I - Problem 1

Danny has 33 more dollars than Raul. Raul has 15 more dollars than Gina. Danny, Raul, and Gina decide to split their money so each of the three will have the same number of dollars. How many dollars will Danny lose?

A. 27
B. 28
C. 6
D. 15
E. 18

Part I - Problem 2

What is the sum of the even integers between 9 and 27?

A. 166
B. 162
C. 158
D. 156
E. 148

Part I - Problem 3

If we choose one digit d from $\{0, 1, 2, 3, \ldots, 9\}$, what is the probability that the number $50d5$ is divisible by 5?

A. $\frac{1}{5}$

B. $\frac{2}{9}$

C. 1

D. $\frac{7}{9}$

E. $\frac{4}{5}$

Part I - Problem 4

For what value of x is

$$2^4 = \left(\frac{1}{4}\right)^x$$

true?

A. -4
B. -2
C. -1
D. 0
E. There is no such value of x.

Part I - Problem 5

An isosceles triangle has side lengths 13 cm, 13 cm, and 24 cm. What is the area of the triangle in square centimeters?

A. $120\ \text{cm}^2$

B. $15\ \text{cm}^2$

C. $30\ \text{cm}^2$

D. $60\ \text{cm}^2$

E. $65\ \text{cm}^2$

Part I - Problem 6

There are 12 teams in a school district competition. If each team plays each other team exactly once, what is the total number of games played in the competition?

A. 63
B. 66
C. 67
D. 68
E. 69

Part I - Problem 7

A grocer stacks apples in the shape of a square pyramid. The bottom layer is a 10×10 square, the top layer is one apple, and the n-th layer is an $n \times n$ square. How many total apples are in the pyramid?

A. $3,025$
B. $1,024$
C. 945
D. 770
E. 385

Part I - Problem 8

The product of the roots of a quadratic function $ax^2 + 2bx + c$ is $\frac{4}{7}$ and the sum of the roots is $\frac{6}{5}$. What is

$$\frac{b+c}{a}$$

A $\frac{7}{10}$

B $-\frac{7}{10}$

C $-\frac{1}{35}$

D $\frac{1}{35}$

E $-\frac{22}{35}$

Part I - Problem 9

The final grade in a physics course is computed by averaging the scores on 10 exams. The scores on the exams can range from 0 to 100, inclusive. Maria's average on the first 3 tests was 88, and her final grade was 74. What was Maria's average on the last 7 tests?

A. 78
B. 75
C. 73
D. 70
E. 68

Part I - Problem 10

The surface area of a cube is $1,350$ square feet. What is the length of the diagonal of a face of the cube?

A. 15 feet

B. $15\sqrt{2}$ feet

C. 30 feet

D. $\frac{15}{2}$ feet

E. 25 feet

Part I - Problem 11

What is the sum of the numbers having the property that each one of them is 6 more than its reciprocal?

A. 10
B. 8
C. 6
D. 4
E. 2

Part I - Problem 12

A dollhouse models a real house at a scale of 1:25. If it takes 625 cans of paint to paint the actual house, how many cans are required to paint the model house?

A. 1 can

B. $\frac{1}{5}$ can

C. 2 cans

D. $\frac{1}{15}$ can

E. $\frac{1}{4}$ can

Part I - Problem 13

Two non-zero real numbers, a and b, satisfy $ab = a - b$.

Find a possible value of $\frac{a}{b} + \frac{b}{a} - ab$.

A -2

B $-\frac{1}{2}$

C $\frac{1}{3}$

D $\frac{1}{2}$

E 2

Part I - Problem 14

The median of the numbers $n+1, n+2, n+3, n+5, n+6, n+9, n+10, n+12, n+15$ is 18. What is the mean of these numbers?

A. 24
B. 22
C. 21
D. 19
E. 18

Part I - Problem 15

The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

A. $2S + 3$
B. $3S + 2$
C. $2S + 6$
D. $3S + 6$
E. $2S + 12$

Part I - Problem 16

For how many positive integers n is $n^2 - 3n + 2$ a prime number?

A. None

B. One

C. Two

D. More than two, but finitely many

E. Infinitely many

Part I - Problem 17

Find k so that the line containing the points $(k, 4)$ and $(-8, 3k)$ is perpendicular to the line with equation $4y = 28 - x$.

A. -4
B. -3
C. -2
D. 2
E. 4

Part I - Problem 18

Evaluate

$$24 \div 3 \times 2 + 12 - 4 \times 3.$$

A. 72
B. 36
C. 16
D. 8
E. 4

Part I - Problem 19

Find 30% of 25% of $\frac{1}{2}$ of 800.

A. 70
B. 63
C. 45
D. 30
E. 10

Part I - Problem 20

If $3 billion is divided equally among 2 million people, how much will each person receive?

A. $15
B. $150
C. $1,500
D. $1.50
E. $0.15

Part I - Problem 21

How many distinct arrangements of the letters in the word BRAIN begin with a vowel?

A. 120
B. 96
C. 72
D. 48
E. 24

Part I - Problem 22

If $x \diamond y = \dfrac{x^2 y^{-2}}{x^2 + y^{-2}}$, what is $\dfrac{1}{3} \diamond \dfrac{1}{2}$?

A. $\dfrac{1}{5}$

B. $\dfrac{5}{13}$

C. $\dfrac{4}{37}$

D. 5

E. $\dfrac{37}{4}$

Part I - Problem 23

Hansel and Gretel want to equally share a circular thin crust pizza of diameter 20 inches. If Hansel's part of the pizza will be circular and cut from the middle, what is the diameter of the hole that Gretel must cut out of the pizza?

A. 10 inches
B. $10\sqrt{2}$ inches
C. 5 inches
D. $5\sqrt{2}$ inches
E. 7 inches

Part I - Problem 24

If $f(x) = 3x^2 + 1$ and $g(x) = x^2 - 2x + 1$, find $f(g(3))$.

A. 148
B. 109
C. 49
D. 13
E. 4

Part I - Problem 25

The diagonals of a rhombus measure 6 and 8. The perimeter of the rhombus is

A. 28
B. 48
C. 24
D. 16
E. 20

Part I - Problem 26

If 4 students can sell 15 cookies in 12 minutes, how many minutes will it take for 6 students to sell 45 cookies (assuming they all sell cookies at the same rate)?

A. 24
B. 20
C. 54
D. 46
E. 32

Part I - Problem 27

What is the maximum value of $\frac{x}{y}$ if $0.005 \leq x \leq 0.5$ and $0.25 \leq y \leq 25$?

A. 5
B. 3
C. 4
D. 2
E. 8

Part I - Problem 28

One circle with radius 2 and two circles with radii 1 are all externally tangent to each other. Find the area of the triangle whose vertices are the centers of the three circles.

A. $2\sqrt{2}$
B. $\sqrt{2}$
C. $3\sqrt{2}$
D. 3
E. 12

Part I - Problem 29

What is the difference between the sum of the first 2024 even positive integers and the sum of the first 2024 odd positive integers?

A. 0
B. 1
C. 2
D. 2024
E. 4048

Part I - Problem 30

The floor function, $\lfloor x \rfloor$, is defined as the largest integer that does not exceed x. For example, $\lfloor 3.1 \rfloor = 3$ and $\left\lfloor \frac{5}{3} \right\rfloor = 1$. Compute

$$\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{16} \rfloor.$$

A. 35
B. 38
C. 40
D. 42
E. 136

Mad Hatter - 15-minute break

Whew! You've reached the end of Part I.

- Please make sure your full name and school name are on your Scantron form.
- Pass your Scantrons in.
- You may leave your belongings here during the break.
- Part II will begin promptly in **15 minutes**.

Mad Hatter - Part II

The rules for this part of the competition are the same as the previous part.

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- Write “**Part 2**” in the box labeled “Exam.”

Ready... Set... Go!

Prepare to restart the *Mad Hatter Marathon!*

Part II - Problem 1

A line with slope 3 intersects a line with slope 5 at point $(10, 15)$. What is the distance between the x-intercepts of these two lines?

A. 2
B. 5
C. 7
D. 12
E. 20

Part II - Problem 2

A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

A. 10%
B. 25%
C. 36%
D. 50%
E. 60%

Part II - Problem 3

What is the digit in the ones place of the number 2024^{2024}?

A. 8
B. 4
C. 0
D. 2
E. 6

Part II - Problem 4

Given that $-4 \leq x \leq -2$ and $2 \leq y \leq 4$, what is the largest possible value of

$$\frac{x+y}{x}?$$

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\frac{1}{2}$

E. 1

Part II - Problem 5

A square has sides of length 10, and a circle centered at one of its vertices has radius 10. What is the area of the union of the regions enclosed by the square and the circle?

A. $200 + 25\pi$
B. $100 + 75\pi$
C. $200 - 25\pi$
D. $100 + 25\pi$
E. $100 + 100\pi$

Part II - Problem 6

The equations $2x - 7 = 3$ and $bx + 12 = -3$ have the same solution x. What is the value of b?

A. 5

B. 3

C. 1

D. -3

E. -5

Part II - Problem 7

The average of 20 numbers is 30, and the average of 30 other numbers is 20. What is the average of all 50 numbers?

A. 22
B. 23
C. 24
D. 25
E. 26

Part II - Problem 8

A positive number x has the property that $x\%$ more than x is equal to $2x$. What is x?

A. 100

B. 99

C. 51

D. 25

E. 24

Part II - Problem 9

On a certain math exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on this exam?

A. 0

B. 1

C. 2

D. 4

E. 5

Part II - Problem 10

What is $(-1)^1 + (-1)^2 + \cdots + (-1)^{2023}$?

A. -1012

B. -1

C. 0

D. 1

E. 1012

Part II - Problem 11

A 2×3 in. rectangle and a 3×4 in. rectangle are contained within a square without overlapping at any point, and the sides of the square are parallel to the sides of the two given rectangles. What is the smallest possible area of the square?

A. 16 square in.
B. 25 square in.
C. 36 square in.
D. 49 square in.
E. 64 square in.

Part II - Problem 12

Real numbers a and b satisfy the equations $3^a = 81^{b+2}$ and $125^b = 5^{a-3}$. What is ab?

A. -60
B. -17
C. 9
D. 12
E. 60

Part II - Problem 13

In a certain land, all Alphas are Betas, all Gammas are Betas, all Deltas are Alphas, and all Gammas are Deltas. Which of the following statements is implied by these facts?

A. All Deltas are Betas and are Gammas.
B. All Betas are Gammas and are Deltas.
C. All Alphas are Gammas and are Deltas.
D. All Gammas are Alphas and are Betas.
E. All Alphas are Deltas, and some Alphas may not be Gammas.

Part II - Problem 14

Some boys and girls are having a car wash to raise money for a class trip to China. Initially 40% of the group are girls. Shortly thereafter two girls leave and two boys arrive, and then 30% of the group are girls. How many girls were initially in the group?

A 4

B 6

C 8

D 10

E 12

Part II - Problem 15

A set of 25 square blocks is arranged into a 5×5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

A. 600
B. 1200
C. 3600
D. 125
E. 100

Part II - Problem 16

If two standard, fair 6-sided dice are rolled, what is the probability that the product of the two numbers rolled is a perfect square?

A. $\frac{1}{36}$

B. $\frac{2}{9}$

C. $\frac{1}{6}$

D. $\frac{1}{4}$

E. $\frac{7}{36}$

Part II - Problem 17

In triangle ABC, $AB = 10$ and $BC = 6$. How many different integer lengths are possible for side AC?

A. 12
B. 11
C. 6
D. 5
E. 2

Part II - Problem 18

Each face of a right square pyramid has a perimeter of 24 cm. What is the volume of the pyramid?

A. 12
B. $6\sqrt{2}$
C. 36
D. 54
E. $36\sqrt{7}$

Part II - Problem 19

If n is an integer for which the greatest common factor of n and 18 is 6 and the least common multiple of n and 9 is 126, what is the value of n?

A. 48

B. 12

C. 36

D. 42

E. 24

Part II - Problem 20

Jesse is making snowballs to have a snowball fight. If Jesse can make 1 snowball every 4 minutes, but 2 snowballs melt every 15 minutes, how long will it take Jesse working continuously to accumulate 21 snowballs? Express your answer to the nearest whole number of hours.

A. 1.5 hours
B. 2 hours
C. 2.5 hours
D. 3 hours
E. 3.5 hours

Part II - Problem 21

Maria compares the price of a new computer at two different stores. Store A offers 20% off the sticker price followed by a $95 rebate, and store B offers 30% off the same sticker price with no rebate. If Maria saves $20 by buying the computer at store A instead of store B, what is the sticker price of the computer?

A. $750

B. $900

C. $1,000

D. $1,200

E. $1,500

Part II - Problem 22

Find the product of the roots of the polynomial

$$3x^3 - 8x^2 - 68x + 48.$$

A. 16
B. 8
C. $\frac{4}{3}$
D. $-\frac{4}{3}$
E. -16

Part II - Problem 23

Solve the equation below for x:

$$n = \sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - \cdots}}}}.$$

A. $x = n^2 + n$
B. $x = n^2 + \sqrt{n}$
C. $x = n^2 - n$
D. $x = n^2 - \sqrt{n}$
E. $x = n^2$

Part II - Problem 24

The product of a certain two-digit number and 5 is equal to the product of 6 with the same two-digit number, but with the digits reversed. What is the quotient of the tens digit and the ones digit in the original number?

A. $\frac{9}{4}$

B. $\frac{4}{9}$

C. $\frac{5}{4}$

D. $\frac{4}{5}$

E. Insufficient information

Part II - Problem 25

The measures of the sides of an isosceles trapezoid have the ratio 3 : 4 : 3 : 6. If the perimeter of the trapezoid is 48 inches, what is its area?

A. 15 square inches

B. $30\sqrt{3}$ square inches

C. $30\sqrt{2}$ square inches

D. $90\sqrt{3}$ square inches

E. $90\sqrt{2}$ square inches

Part II - Problem 26

A basketball player makes 70% of her free throws. What is the probability that she will make both of her next two free throws?

- A $\frac{7}{10}$
- B $\frac{21}{100}$
- C $\frac{49}{100}$
- D $\frac{51}{100}$
- E $\frac{79}{100}$

Part II - Problem 27

Pat drove to work on Monday at an average speed of 40 mph and arrived one minute late. She left for work at the same time on Tuesday and drove at an average speed of 45 mph. If she arrived one minute early on Tuesday, how far does she have to drive to get to work?

A. 12 miles
B. 9 miles
C. 8 miles
D. 15 miles
E. 6 miles

Part II - Problem 28

The sum of three numbers, a, b, and c, is 99. If we increase a by 6, decrease b by 6, and multiply c by 5, the three resulting numbers are equal. What is b?

A. $b = 9$
B. $b = 18$
C. $b = 21$
D. $b = 39$
E. $b = 51$

Part II - Problem 29

Every student in the senior class is taking history or science, and 85 of them are taking both. If 106 seniors are taking history and 109 seniors are taking science, how many students are in the senior class?

A. 215
B. 195
C. 130
D. 191
E. 194

Part II - Problem 30

Chris' running schedule dictates that he never runs more than two days in a row, and that he never goes more than two days in a row without running. In two weeks, what is the difference between the largest number of days he could have run and the smallest number of days?

A 2

B 3

C 4

D 5

E 6

Mad Hatter - Done!

You made it!

- Please make sure your full name and school name are on your Scantron form.
- Pass your Scantron in.
- Please take your belongings with you.
- **Program change:** Instead of board and card games, there will be a science and math carnival outside the Science 2 Building starting at noon.
- The awards ceremony will begin at **2:45pm**. If there are any ties, you have to be present to win the tiebreaker. See you there!

Part I

1	A	6	B	11	C	16	B	21	D	26	A
2	B	7	E	12	A	17	A	22	C	27	D
3	C	8	C	13	E	18	C	23	B	28	A
4	B	9	E	14	D	19	D	24	C	29	D
5	D	10	B	15	E	20	C	25	E	30	B

Part II

1	A	6	D	11	B	16	B	21	A	26	C
2	C	7	C	12	E	17	B	22	E	27	A
3	E	8	A	13	D	18	E	23	A	28	E
4	D	9	B	14	C	19	D	24	C	29	C
5	B	10	B	15	A	20	D	25	E	30	D