Department Qualifying Exam (*Traditional M.A. Students*)
Algebra Syllabus

Note: The exam consists of two sections (abstract algebra and linear algebra) with 8 questions per sections. Students must answer 5 questions per section. The exam topics are normally covered in Math 151 and Math 152.

Topics: The student is expected to know at least the following topics. Although the list is reasonably comprehensive, it is just an indication of all topics that could be covered in the exam. It is the student's responsibility to prepare adequately for the exam.

Part I (Abstract Algebra):

1. equivalence relations;
2. basic properties of the *gcd* and *lcm* of two integers;
3. groups: both additive and multiplicative, Abelian and non-Abelian. Must know how to check a set (with an operation) is a group, examples and properties of well-known groups ($\mathbb{Z}_n$, D_n, S_n, A_n, Q_8, etc);
4. the order of a group, and of an element in a group; Lagrange's theorem, Euler's theorem,
5. cyclic groups;
6. permutation groups (S_n and A_n); multiplication, even and odd permutations;
7. subgroups: examples of well-known subgroups, checking for a set to be a subgroup of a given group, cosets of a subgroup, index of a subgroup;
8. normal subgroups and factor (quotient) groups;
9. group homomorphisms; kernel and image of a group homomorphism, isomorphisms;
10. first isomorphism theorem and corollaries (the other homomorphism theorems);
11. direct product/sum of groups;
12. rings: must know how to check a set, with two given operations, is a ring (note that for this exam all rings have a unity), examples and properties of well-known rings ($\mathbb{Z}$, $\mathbb{Z}_n$, polynomial rings, matrices, etc), invertible elements, zero-divisors;
13. $\mathbb{Z}$ and $\mathbb{Z}_n$, Chinese remainder theorem;
14. subrings; left, right and two-sided ideals;
15. ring homomorphisms; kernel and image of a ring homomorphism, isomorphisms;
16. basic properties of integral domains, division rings and fields.

Part II (Linear Algebra):

1. vector spaces over $\mathbb{R}$ and $\mathbb{C}$; know how to work with the basic examples of vector spaces: $\mathbb{R}^n$, $\mathbb{C}^n$, matrices, $\mathcal{P}_n$, function spaces, etc.;
2. spanning sets, linearly independent sets, bases; change of basis; dimension;
3. linear transformations; must be able to find a matrix for a linear map in any given pair of bases;

4. kernel and range of a linear map; nullity and rank; dimension formula;
5. matrices: symmetric, diagonal, elementary, echelon form, upper/lower diagonal, block matrices, etc.; matrix algebra: inverses, transposes, etc.;
6. determinants; relation with the invertibility of a matrix;
7. how to solve systems of equations using elementary operations; know how homogeneous systems of equations yield subspaces;
8. eigenvalues (for this exam only real eigenvalues will be considered), eigenvectors, eigenspaces;
9. diagonalization of matrices, including with (maybe) repeated eigenvalues;
10. inner product spaces; Gram-Schmidt orthonormalization process.

Suggested References:

- *Contemporary Abstract Algebra* by Joseph A. Gallian
- *A First Course in Abstract Algebra* by John B. Fraleigh
- *Abstract Algebra* by I.N. Herstein
- *Abstract Algebra* by John Beachy and William Blair
- *A First Course in Abstract Algebra* by Joseph Rotman
- *Linear Algebra* by K. Hoffman and R. Kunze
- *Elementary Linear Algebra* by Ron Larson and David Falvo
- *Linear Algebra* by Steven J. Leon.
- *Elementary Linear Algebra* by David Kolman and Bernard Hill.

Department Qualifying Exam (*Traditional M.A. Students*)
Analysis Syllabus

Note: The exam will consist of 12 questions. Students must answer 8 of these questions. The exam topics are normally covered in Math 77 and Math 171.

Topics: The student is expected to know at least the following topics. Although the list is reasonably comprehensive, it is just an indication of all topics that could be covered in the exam. It is the student's responsibility to prepare adequately for the exam.

1. natural numbers, the Archimedean Property;
2. rational numbers, rational zeros theorem, irrational numbers;
3. the real numbers, Completeness Axiom, density of the rational numbers in the real numbers;
4. sequences of real numbers, Cauchy sequences;
5. subsequences, the Bolzano-Weierstrass theorem;
6. infinite series, ratio and root tests, integral test, alternating series test;
7. limits of functions, continuity, uniform continuity;
8. Intermediate Value Theorem;
9. extension of functions, continuous extensions;
10. differentiability of a function of one real variable;
11. the Mean Value Theorem, the mean value theorem for derivatives;
12. L'Hospital's rule;
13. partitions, the Darboux/Riemann integral;
14. integrable functions, non-integrable functions of one real variable;
15. vectors, dot product, cross product and applications to area and volume calculations;
16. equations of lines, planes, normal lines, distances between lines/planes;
17. arc length, curvature;
18. functions of several variables;
19. continuity of functions of several variables;
20. partial and directional derivatives, the gradient of a function;
21. extreme value problems;
22. Lagrange multipliers and applications;
23. multiple integrals in Euclidean, polar, and spherical coordinates; area and volume calculations;
24. change of variables in multiple integrals;
25. vector fields, curl and divergence;

26. surface integrals;
27. Stokes' theorem;
28. line integrals;
29. Green's theorem;
30. Fundamental Theorem of Line Integrals.

Suggested References:

- *Elementary Analysis: The Theory of Calculus* by Kenneth A. Ross
- *An Introduction to Analysis* by William R. Wade
- *Introduction to Analysis* by Edward D. Gaughan
- *Real Analysis: A Constructive Approach* by Mark Bridger
- *Essential Calculus – Early Transcendentals* by J. Stewart (or any calculus book covering vector calculus)