

CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10
PART I

April 21st, 2012

2. A triangle has sides of length x , y , and z . Which of the following must be false?

- (a) $x + y = 2z$
- (b) $x + z = y + 2$
- (c) $y + z > x$
- (d) $x + y < z$

1. What is

$$8 + 6 - 2 \times 3 \div 2?$$

- (a) 11
- (b) 18
- (c) 4
- (d) 9

3. Sam ordered pizzas with 32 slices for his party. How many slices will each person receive if 40 people showed up to his party, and they share the pizza equally?

- (a) $\frac{1}{2}$ slice
- (b) $\frac{4}{5}$ slice
- (c) 80% of a slice
- (d) Both (b) and (c) are correct

4. Solve the following system of equations

$$4r - 3s = 20$$

$$2r = 3s + 5.$$

(a) $r = \frac{15}{2}, s = \frac{10}{3}$

(b) $r = 15, s = \frac{10}{3}$

(c) $r = \frac{5}{2}, s = 0$

(d) no solution

6. Find the equation of the line that passes through the point $(2, -1)$ and is perpendicular to the line $2x - 3y = 5$.

(a) $y = \frac{3}{2}x - 3$

(b) $y = \frac{2}{3}x - \frac{7}{3}$

(c) $y = -\frac{3}{2}x + 2$

(d) $y = -\frac{2}{3}x + \frac{1}{3}$

5. A quarterback throws a pass from the five-yard line, 20 yards from the sideline. The pass is caught by a wide receiver on the 45-yard line, 50 yards from the same sideline. What was the total distance the football traveled?

(a) 50 yards

(b) 45 yards

(c) 30 yards

(d) 25 yards

7. A landscaper wants to reshape a square tee area for the ninth hole of a golf course. The new tee area is to have one side two feet longer and the adjacent side six feet longer than the original tee. The new tee has 204 square feet more area than the original tee. What are the dimensions of the original tee?

(a) 24.5 feet by 24.5 feet

(b) 24 feet by 24 feet

(c) 30 feet by 32 feet

(d) Not enough information is given

8. Simplify

$$\left(\frac{6x^{-8}y^{-3}z^2}{2x^{-5}y^6z}\right)^{-2}$$

- (a) $\frac{9x^6}{y^{18}z^2}$
- (b) $\frac{x^6y^{18}}{9z^2}$
- (c) $\frac{1}{9x^5y^{11}z}$
- (d) $\frac{y^{18}}{9x^{26}z^2}$

10. If $\frac{x}{2} - \frac{x}{6}$ is an integer, which of the following statements must be true?

- (a) x is odd
- (b) x is even
- (c) x is a multiple of 6
- (d) x is a multiple of 3

9. Let $f(x) = x^2 + 6$ and $g(x) = 2x^2$. What is $f(g(x))$?

- (a) $4x^4 + 6$
- (b) $4x^2 + 12$
- (c) $2x^4 + 12x^2$
- (d) $2x^4 + 24x^2 + 72$

11. Two hundred tickets were sold for a school play. Some tickets cost \$10 apiece, and the others cost \$5 apiece. If total ticket sales were \$1,750, how many of the more expensive tickets were sold?

- (a) 150
- (b) 100
- (c) 75
- (d) 50

12. In a class of 10 boys and 15 girls, the boys' average score on the final exam was 80 and the girls' average score was 90. What was the average score of the entire class?

- (a) 83
- (b) 84
- (c) 85
- (d) 86

13. Judy is now twice as old as Adam, but six years ago, she was five times as old as he was. How old is Judy now?

- (a) 10
- (b) 16
- (c) 20
- (d) 24

14. If x and y are integers such that $x^3 = y^2$, which of the following could NOT be the value of y ?

- (a) -1
- (b) 1
- (c) 8
- (d) 16

15. The positive difference between the squares of any two consecutive numbers must be

- (a) the square of an integer.
- (b) an even number.
- (c) an odd number.
- (d) a prime number.

16. What is the probability that the sum of two different single-digit prime numbers will be prime?

- (a) 0
- (b) $\frac{1}{2}$
- (c) $\frac{1}{3}$
- (d) $\frac{1}{6}$

18. M college students agree to rent an apartment for D dollars per month, sharing the rent equally. If the rent is increased by \$100, what amount must each student contribute?

- (a) $\frac{M + 100}{D}$
- (b) $\frac{D}{M} + 100$
- (c) $\frac{M}{D + 100}$
- (d) $\frac{D + 100}{M}$

17. What is the greatest possible value of integer n if $6^n < 10000$?

- (a) 4
- (b) 5
- (c) 6
- (d) 7

19. If $@$ is an operation defined so that

$$x@y = \frac{x^2 - 2xy + y^2}{x^2 - y^2}$$

and $x^2 - y^2 \neq 0$, then what is $c@d$ if $c = 5d$?

- (a) $-\frac{3}{2}$
- (b) $-\frac{2}{3}$
- (c) $\frac{2}{3}$
- (d) $\frac{3}{2}$

20. An antique dealer usually charges 20% more than her purchase price for any vase sold in her store. During a clearance sale, all items are marked 10% off. If she sells a vase during the clearance sale, what percent of the purchase price was her profit (sale price less purchase price)?

- (a) 8%
- (b) 9%
- (c) 10%
- (d) 11%

22. One side of a rectangle is the diameter of a circle and the opposite side is tangent to the circle. Find the difference between the area of the rectangle and the area of the circle in terms of r , the radius of the circle.

- (a) $4r^2 - \pi r^2$
- (b) $\pi r^2 - 4r^2$
- (c) $2r^2 - \pi r^2$
- (d) $\pi r^2 - 2r^2$

21. Two boats leave the same dock at the same time, one traveling due east at 10 miles per hour and the other traveling due north at 24 miles per hour. How many miles apart are the boats after 3 hours?

- (a) 68
- (b) 72
- (c) 78
- (d) 110

23. A closed rectangular box with a square base is 5 inches in height. If the volume of the box is 45 cubic inches, determine the box's surface area.

- (a) 81 square inches
- (b) 78 square inches
- (c) 66 square inches
- (d) 45 square inches

24. A flagpole casts a shadow 16 feet long. At the same time, a pole 9 feet high casts a shadow 6 feet long. How tall is the flagpole?

- (a) 24 feet
- (b) 20 feet
- (c) 19 feet
- (d) 18 feet

26. A manufacturer designs a pyramid-shaped container 15 cm high with a square base measuring 10 cm on a side. After testing the design, the manufacturer decides to increase the length and width of the base by 4 cm. By how much does the volume of the package increase?

- (a) 2940 cubic cm
- (b) 980 cubic cm
- (c) 500 cubic cm
- (d) 480 cubic cm

25. In an isosceles triangle, the largest angle is equal to the sum of the two smallest angles. What is the measure of the largest angle?

- (a) 120°
- (b) 100°
- (c) 90°
- (d) 60°

27. The sum of three consecutive even numbers is 90. What is the largest of the three numbers?

- (a) 26
- (b) 28
- (c) 30
- (d) 32

28. A bag contains 12 red, 3 blue, 6 green, and 4 yellow marbles. If a marble is drawn from the bag at random, what is the probability that the marble will be either blue or yellow?

- (a) 40%
- (b) 28%
- (c) 25%
- (d) 16%

30. In triangle ABC , the measure of $\angle A$ is 30° and the measure of $\angle B$ is 60° . If \overline{AB} is 16 units long, what is the area, in square units, of triangle ABC ?

- (a) 16
- (b) $16\sqrt{3}$
- (c) $32\sqrt{3}$
- (d) 256

29. If

$$\frac{2}{x} \geq \frac{1}{7},$$

what is the largest possible value of x ?

- (a) $\frac{1}{2}$
- (b) 7
- (c) 14
- (d) 28

31. If $6a^5b^7 < 0$, which of the following must be true?

- (a) ($a > 0$ and $b > 0$) or ($a < 0$ and $b < 0$)
- (b) ($a > 0$ and $b < 0$) or ($a < 0$ and $b > 0$)
- (c) $a < b$
- (d) $a > b$

32. What is the smallest possible value for the product of two real numbers that differ by 8?

- (a) -16
- (b) -4
- (c) 8
- (d) 9

34. A bus line charges \$5 each way to ferry a passenger between the hotel and an archaeological dig. On a given day, the bus line has a capacity to carry 255 passengers from the hotel to the dig and back. If the bus line runs at 90% of capacity, how much money did the bus line take in that day?

- (a) \$1,147.50
- (b) \$2,295
- (c) \$2,550
- (d) \$2,625

33. Four carpenters built an average of 42 chairs last week. If no chairs were left uncompleted, and if Peter, who built 50 chairs, built the greatest number of chairs, what is the **least** number of chairs one of the other carpenters could have built, if no carpenter built a fractional number of chairs.

- (a) 18
- (b) 19
- (c) 20
- (d) 33

35. Consider all positive integer values a and b such that $ab = 8$. For how many values does there exist a positive integer c that satisfies both $2^a = c$ and $c^b = 256$?

- (a) Infinitely many
- (b) 4
- (c) 2
- (d) 0

36. A sphere is inscribed in a cube with a diagonal of $3\sqrt{3}$ ft. In feet, what is the diameter of the sphere?

- (a) $3\sqrt{3}$
- (b) 2
- (c) $2\sqrt{2}$
- (d) 3

37. Find all solutions of $|x - 5| > 2$.

- (a) $x > 7$
- (b) $x < 3$
- (c) $x < -3$
- (d) $x > 7$ or $x < 3$

38. The gravitational attraction between two bodies varies inversely as the square of the distance between them. If the force of attraction is 64 pounds when the distance between the bodies is 9 feet, what is the force in pounds when they are 24 feet apart?

- (a) 9
- (b) 24
- (c) 729
- (d) 5184

39. The diameter of each tire on a motorcycle is 21 inches. About how many revolutions does one of these tires make when the motorcycle travels 550 feet (6,600 inches) without slipping?

- (a) 100
- (b) 85
- (c) 50
- (d) 24

40. After N chocolate bars are divided equally among 6 children, 3 bars remain. How many bars would remain if $N + 4$ chocolate bars were divided equally among the 6 children?

- (a) 0
- (b) 1
- (c) 2
- (d) 4

- 17 B
- 18 D
- 19 C
- 20 A
- 21 C
- 22 C
- 23 B
- 24 A
- 25 C
- 26 D
- 27 D
- 28 B
- 29 C
- 30 C
- 31 B
- 32 A
- 33 C

Solutions

- 1 A
- 2 D
- 3 D
- 4 A
- 5 A
- 6 C
- 7 B
- 8 B
- 9 A
- 10 D
- 11 A
- 12 D
- 13 B
- 14 D
- 15 C
- 16 C

- 34 B
- 35 B
- 36 D
- 37 D
- 38 A
- 39 A
- 40 B

CSU FRESNO MATHEMATICS FIELD DAY

MAD HATTER MARATHON 9-10
PART II

April 21st, 2012

2. What is the remainder when $x^{51} + 51$ is divided by $x + 1$?

- (a) 1
- (b) 49
- (c) 50
- (d) 51

1. What is the ones digit of 24^{2012} ?

- (a) 0
- (b) 2
- (c) 6
- (d) 8

3. Simplify the expression below, where a , b , and c are distinct real numbers.

$$\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)}$$

- (a) 0
- (b) 1
- (c) $\frac{(x-a)(x-b)(x-c)}{(a-b)(b-c)(c-a)}$
- (d) $\frac{3x^2}{(a-b)(b-c)(c-a)}$

4. The sum of the third and fourth terms in a sequence of consecutive integers is 47. What is the sum of the first five terms of the sequence?

- (a) 115
- (b) 93
- (c) 90
- (d) 72

5. Which number below is the largest?

- (a) 6^{100}
- (b) 5^{200}
- (c) 4^{300}
- (d) 3^{400}

6. The area of the largest triangle that can be inscribed in a semi-circle of radius r is

- (a) $2r^3$
- (b) $2r^2$
- (c) r^2
- (d) $\frac{1}{2}r^2$

7. $\sqrt{9 + 4\sqrt{2}} =$

- (a) $1 + 2\sqrt{2}$
- (b) $3 + 2\sqrt{2}$
- (c) $3 + 2\sqrt{2}$
- (d) $2 + 3\sqrt{2}$

8. Assume that a and b are integers greater than one. In base a , b^2 is written as 10. Then a^2 , when written in base b , is

- (a) 100
- (b) 101
- (c) 1010
- (d) 10000

9. For a certain positive integer n , $5n + 16$ and $8n + 29$ have a common factor larger than one. That common factor is

- (a) 10
- (b) 17
- (c) 13
- (d) 12

10. Let $f(x)$ be a quadratic polynomial such that $f(2) = -3$ and $f(-2) = 21$. What is the coefficient of x in $f(x)$?

- (a) -6
- (b) -4
- (c) 0
- (d) 4

11. What is the value of the constant term in the expansion of

$$\left(\frac{x^2}{3} - \frac{3}{x^2}\right)^6?$$

- (a) -3^6
- (b) -20
- (c) 0
- (d) $\left(\frac{1}{3}\right)^6$

12. For what values of p does $4x^2 + 4px + 4 - 3p = 0$ have two distinct real roots?

- (a) $-1 < p < 4$
- (b) $-4 < p < 1$
- (c) $p < -1$ or $p > 4$
- (d) $p < -4$ or $p > 1$

14. A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle?

- (a) $\frac{\sqrt{\pi}}{2}$
- (b) $\sqrt{\pi}$
- (c) π
- (d) π^2

13. If it were two hours later, it would be half as long until midnight as it would be if it were an hour later. The time now is

- (a) 7:00 p.m.
- (b) 8:00 p.m.
- (c) 8:30 p.m.
- (d) 9:00 p.m.

15. Square $EFGH$ has one vertex on each side of square $ABCD$. Point E is on \overline{AB} with $AE = 7 \cdot EB$. What is the ratio of the area of $EFGH$ to the area of $ABCD$?

- (a) $\frac{49}{64}$
- (b) $\frac{25}{32}$
- (c) $\frac{7}{8}$
- (d) $\frac{5\sqrt{2}}{8}$

16. The first 2012 positive integers are each written in base 3. How many of these base-3 representations are palindromes? (A palindrome is a number that reads the same forward and backward.)

- (a) 100
- (b) 101
- (c) 102
- (d) 103

18. Suppose that a and x are two positive real numbers for which $\log_a x + \log_x a = 3$. What is $(\log_a x)^2 + (\log_x a)^2$?

- (a) 2
- (b) 6
- (c) 7
- (d) 9

17. An urn contains N black marbles and N white marbles. Three marbles are chosen from the urn randomly and without replacement. What is N if the probability is $\frac{1}{12}$ that all three chosen are white?

- (a) 5
- (b) 6
- (c) 7
- (d) 8

19. Suppose that the sum of k consecutive integers is an even integer divisible by k and that the smallest of the k consecutive integers is even. Which of the following must be true about k ?

- (a) $k + 3$ is divisible by 4
- (b) $k + 2$ is divisible by 4
- (c) $k + 1$ is divisible by 4
- (d) k is divisible by 4

20. A convex polygon with n sides has all angles equal to 150° , with the possible exception of one angle. Which of the following intervals contains all possible values of n ?

- (a) $6 \leq n \leq 12$
- (b) $7 \leq n \leq 12$
- (c) $7 \leq n \leq 13$
- (d) $8 \leq n \leq 12$

22. Three boys Alan, Bob, and Charlie, can work separately or together on a job. It take Alan working alone twice as long as it takes Bob working alone. It takes Alan four times as long as Bob and Charlie working together to do the job. Also, it takes Alan two hours longer to do the job than it takes all three boys working together. How many hours does it take Bob to do the job alone?

- (a) $\frac{5}{2}$
- (b) $\frac{5}{4}$
- (c) $\frac{2}{5}$
- (d) $\frac{4}{5}$

21. You run two miles at 8 miles per hour and three more miles at 6 miles per hour. What is your average speed for the five mile run?

- (a) $6\frac{2}{3}$ mph
- (b) 6.75 mph
- (c) 7 mph
- (d) 7.5 mpg

23. If 1, x , y , 27 form the start of a geometric progression, $y =$

- (a) $\sqrt[3]{9}$
- (b) $\sqrt{3}$
- (c) 3
- (d) 9

24. Which of the following statements are incorrect?

- I. Doubling the base of a rectangle doubles its area.
- II. Doubling the radius of a circle doubles its area.
- III. Doubling a given number may make it less than it originally was.

- (a) I. is incorrect.
- (b) II. is incorrect.
- (c) III. is incorrect.
- (d) Both II. and III. are incorrect.

26. For how many positive integers less than or equal to 100 is the sum of the digits less than or equal to six?

- (a) 7
- (b) 19
- (c) 27
- (d) 28

25. Mary has sixteen coins that total \$3.00. If she only has nickels and quarters, how many quarters does she have?

- (a) 4
- (b) 5
- (c) 11
- (d) This problem has no solution.

27. What is the value of the product

$$2^{\frac{1}{2}} \cdot 2^{\frac{1}{4}} \cdot \dots \cdot 2^{\frac{1}{2^n}} \cdot \dots?$$

- (a) 0
- (b) 1
- (c) 2
- (d) 4

28. There are four roads connecting town A to town B , and three roads from town B to town C . How many roads are there from town A to town C that pass through town B ?

- (a) 12
- (b) 7
- (c) 4
- (d) 3

30. Suppose hops, skips, and jumps are specific units of length. If b hops equals c skips, d jumps equals e hops, and f jumps equals g meters, then one meter equals how many skips?

- (a) $\frac{bdg}{cef}$
- (b) $\frac{cdf}{beg}$
- (c) $\frac{cdg}{bef}$
- (d) $\frac{cef}{bdg}$

29. Find all solutions of

$$|x + 1| + 2|x - 2| \leq 6.$$

- (a) $-1 \leq x \leq 2$
- (b) $0 \leq x \leq 1$
- (c) $-1 \leq x \leq 3$
- (d) $x \leq -1$ or $x \geq 2$

31. Three tennis balls fit snugly into a cylindrical can. The balls touch the side, top, and bottom of the can. What is the ratio of the volume of the balls to the volume of the space in the can that is around the tennis balls?

- (a) 1.414 to 1
- (b) 1.732 to 1
- (c) 2 to 1
- (d) 3 to 2

32. There are 29 people in a room. Of these, 11 speak Spanish, 24 speak English, and 3 speak neither Spanish nor English. How many in the room speak both Spanish and English?

- (a) 3
- (b) 4
- (c) 6
- (d) 9

34. On the blackboard, the math teacher wrote a polynomial $f(x)$ with integer coefficients and said, "Today is my son's birthday. When his age A is substituted for x , then $f(A) = A$. You will also note that $f(0) = P$ where P is a prime number greater than A ." How old is the teacher's son?

- (a) 1
- (b) 2
- (c) 4
- (d) 6

33. A student has a median test score of 81 on 5 tests. She can only find four of the test scores; they are 81, 67, 79, and 92. Which of the following could not be the fifth score?

- (a) 25
- (b) 81
- (c) 100
- (d) None of them could be the score.

35. Let x and y be positive numbers. If $\frac{x+y}{x} = \frac{x}{y}$, find the ratio $\frac{y}{x}$.

- (a) $\frac{2}{3}$
- (b) $\frac{\sqrt{5}-1}{2}$
- (c) $\frac{3-\sqrt{5}}{2}$
- (d) $\frac{3}{5}$

36. The equation $3^{x^2} = 81^{2x-3}$ has two solutions. What is their sum?

- (a) -2
- (b) 2
- (c) 6
- (d) 8

38. Lisa has a collection of 50 songs that are each 3 minutes in length and 50 songs that are each 5 minutes in length. What is the maximum number of songs from her collection that she can play in 5 hours?

- (a) 60
- (b) 70
- (c) 80
- (d) 90

37. The ratio of dark to light squares is $1.0\overline{16}$ on a square game board beginning with dark and alternating to light. What are the dimensions of the board?

- (a) 9×9
- (b) 11×11
- (c) 13×13
- (d) 15×15

39. If $(\sqrt{3}^x)(\sqrt{27}^x) = \sqrt{3}$, what is the value of x ?

- (a) $\frac{1}{8}$
- (b) $\frac{1}{6}$
- (c) $\frac{1}{4}$
- (d) $\frac{1}{2}$

40. Find the maximum value of k so that the inequality $\sqrt{x} + \sqrt{6-x} \geq k$ has a solution.

- (a) $\sqrt{6} - \sqrt{3}$
- (b) $\sqrt{6} + \sqrt{3}$
- (c) $\sqrt{6}$
- (d) $2\sqrt{3}$

- 17 A
- 18 C
- 19 A
- 20 D
- 21 A
- 22 B
- 23 D
- 24 B
- 25 C
- 26 D
- 27 C
- 28 A
- 29 C
- 30 D
- 31 C
- 32 D
- 33 A

Solutions

- 1 C
- 2 C
- 3 B
- 4 A
- 5 D
- 6 C
- 7 A
- 8 D
- 9 B
- 10 A
- 11 B
- 12 D
- 13 D
- 14 B
- 15 B
- 16 A

- 34 A
- 35 B
- 36 D
- 37 B
- 38 C
- 39 C
- 40 D