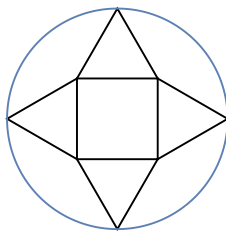


2017
Leap Frog Relay Grades 11-12
Part I

No calculators allowed

Correct Answer = 4, Incorrect Answer = -1, Blank = 0

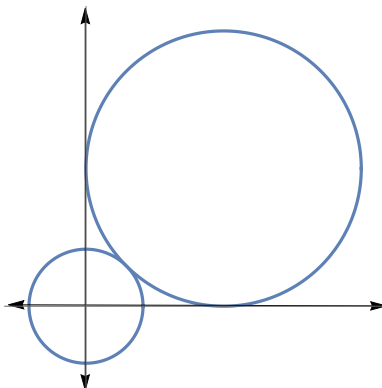
- If r_1 and r_2 are the two real number solutions to the equation $x^2 + x = 2017$, then $(r_1 + r_2)^{2017} = \underline{\hspace{2cm}}$.
 - 0
 - 1
 - 2^{2017}
 - -2^{2017}
 - None of these
- The central square is sharing its sides with 4 equilateral triangles, and the combined figure is inscribed in the circle as pictured below.



What is the ratio of circle area to square area?

- $\sqrt{6}\pi$
- $\pi \left(1 + \frac{\sqrt{3}}{2}\right)$
- 2π
- $\pi (1 + \sqrt{3})$
- None of these

3. If you triple the radius of a circle, then the resulting *percentage* increase in circle area is _____.
- (a) 300% (b) 600%
- (c) 800% (d) 900%
- (e) None of these
4. In the figure below, the smaller circle is centered at the origin and has radius equal to a , while the larger circle is mutually tangent to the smaller circle and the two coordinate axes, with radius equal to b . Then, $b/a =$ _____.



- (a) $\frac{3}{2}$ (b) 2
- (c) $1 + \sqrt{2}$ (d) $\frac{5}{2}$
- (e) None of these
5. If $\log_{4034} 2 = a$, then $\log_{2017} 4034 =$ _____.
- (a) $\frac{1}{a}$ (b) $\frac{1}{1+a}$
- (c) $\frac{1}{2a}$ (d) $\frac{2}{1+a}$
- (e) None of these

6. If $\sqrt[3]{4} \cdot \sqrt[4]{x} = 2 \sqrt[12]{32}$, then $x =$ _____.
- (a) 64 (b) 8
(c) 4 (d) 32
(e) None of these
7. If $\sin(x + \pi) = \sin(x + \pi/2)$ and $0 < x < \pi$ (x is measured in radians), then $x =$ _____.
- (a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$
(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$
(e) None of these
8. Suppose N is the smallest integer larger than 1 such that when divided by *every* $k = 2, 3, \dots, 10$, the resulting remainder is 1. Then, ...
- (a) $500 < N < 1000$ (b) $1000 < N < 1500$
(c) $1500 < N < 2000$ (d) $2000 < N < 2500$
(e) None of these
9. Define a function f on *positive integers* by

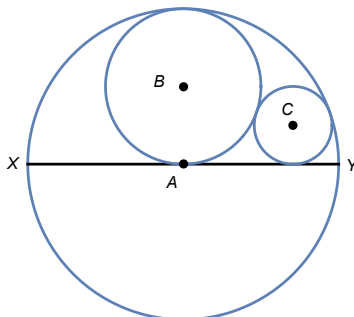
$$f(x) = \begin{cases} x/2 & \text{if } x \text{ is even,} \\ 3x + 1 & \text{if } x \text{ is odd.} \end{cases}$$

How many (integer) solutions are there to the equation

$$f(x) + f(x + 1) = 2017?$$

- (a) 0 (b) 1
(c) 2 (d) 3
(e) None of these

10. Let's label the three circles pictured below by their respective centers $A, B,$ and C . Circle B is tangent to circle A and goes through the center point A and is tangent to the diameter \overline{XY} of circle A . Circle C is mutually tangent to circles A and B and the diameter \overline{XY} . If the radius of circle A is R , then the radius of circle C is _____.



- (a) $\frac{R}{2\sqrt{2}}$ (b) $\frac{R}{4}$
 (c) $\frac{R}{2 + \sqrt{2}}$ (d) $\frac{R}{4\sqrt{2}}$
 (e) None of these

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11. The positive real number solution to the equation

$$\frac{x}{2017} - \frac{2017}{x} = 1$$

is ...

(a) $x = 2017(\sqrt{5} + 1)$

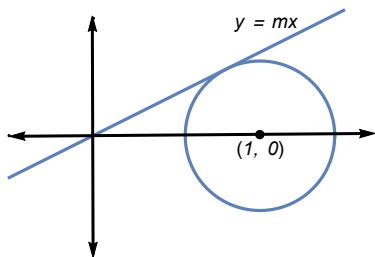
(b) $x = 2017(\sqrt{5} - 1)$

(c) $x = \frac{\sqrt{5} - 1}{2017}$

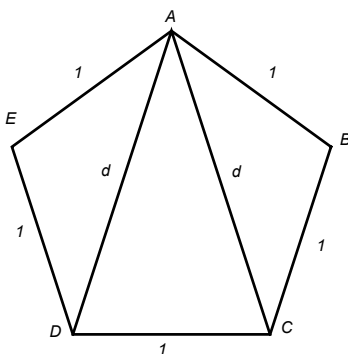
(d) $x = \frac{\sqrt{5} + 1}{2017}$

(e) None of these

12. In the figure below, the circle centered at the point $(1, 0)$ is tangent to the line $y = mx$, where $m > 0$. Then, the radius of the circle is _____.

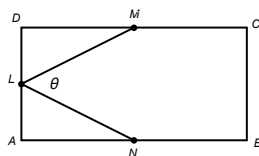


- (a) $\frac{1}{\sqrt{m^2 + 1}}$ (b) $\frac{m + 1}{\sqrt{m^2 + 1}}$
 (c) $\frac{m^2}{\sqrt{m^2 + 1}}$ (d) $\frac{m}{\sqrt{m^2 + 1}}$
 (e) None of these
13. The pentagon $ABCDE$ pictured below is a *regular* pentagon with all five side lengths equal to 1. Let $d = AC = AD$. Then, $d =$ _____.



- (a) $\frac{\sin 108^\circ}{\sin 36^\circ}$ (b) $\frac{2 \sin 108^\circ}{\sin 36^\circ}$
 (c) $\frac{\sin 108^\circ}{2 \sin 36^\circ}$ (d) $\frac{2 \sin 108^\circ}{3 \sin 36^\circ}$
 (e) None of these

14. How many multiples of 2017 with the units digit equal to 1 are there between 0 and 20,172,017?
- (a) 999 (b) 1000
(c) 1001 (d) 1002
(e) None of these
15. The solution to the inequality $-1 \leq |x - 2| - |x - 4| \leq 1$ is in the form $a \leq x \leq b$. Then, $a + b =$ _____.
- (a) 4 (b) 5
(c) 6 (d) 7
(e) None of these
16. A one percent increase in the diagonal length of a square results in what percentage increase in its area?
- (a) 1.99% (b) 2%
(c) 2.01% (d) 2.02%
(e) None of these
17. In the rectangle $ABCD$ pictured below, $AB = DC = a$, $AD = BC = b$, and L , M , N are the respective midpoints of \overline{AD} , \overline{DC} , \overline{AB} . Let $\theta = m\angle MLN$. Then, $\cos \theta =$ _____.



- (a) $\frac{a - b}{a + b}$ (b) $\frac{a}{b}$
(c) $\frac{a^2 - b^2}{a^2 + b^2}$ (d) $\frac{a^2 + b^2}{a^2 - b^2}$
(e) None of these

18. Lenny has \$5.85 in nickels, dimes and quarters in his pocket. Assuming he has 52 coins, what is the least number of nickels he could have?
- (a) 1 (b) 2
(c) 3 (d) 4
(e) None of these
19. If you divide 2017 by 20, there results the remainder 17. Find the number of integers m larger than 17 (and smaller than 2017) for which if you divide 2017 by m , there results the remainder 17.
- (a) 11 (b) 12
(c) 13 (d) 14
(e) None of these
20. Suppose a, b, c, d are positive real numbers. Then,

$$\log_{(a^b)}(c^d) = \text{_____}.$$

- (a) $\frac{d \log_a c}{b}$ (b) $\frac{d \log_a c}{\log_a b}$
(c) $\frac{d \log_a c}{\log_b a}$ (d) $\frac{d \log_b c}{\log_a b}$
(e) None of these