

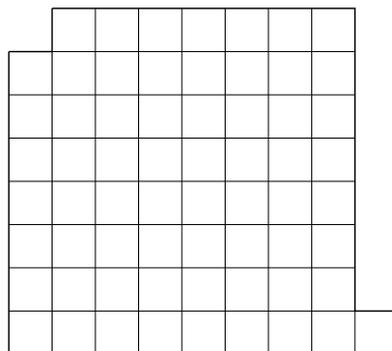
2015 Sonia Kovalevsky Day Problem Solving Competition Due by 11/29/15 at 11:59 pm

Instructions: In this competition, you will:

1. Solve the following problems with proof. Your proofs must be mathematically precise and correct. You are strongly encouraged to research textbooks and other sources for information but you are **NOT** allowed to seek help from teachers or other individuals.
2. Typeset your solutions in your favorite text editor or handwrite your solutions on paper. Send your typeset document or scan your solution paper(s) and send it via e-mail with subject line **SK Day Problem Solving Contest** to **sk2015psc@gmail.com** by the deadline listed above. **Your solution file must be in .jpg or .pdf format only.**
3. Your submission **MUST** include the following information:
 - (a) Your name
 - (b) Your e-mail address
 - (c) Your school name
 - (d) Your grade level
 - (e) Your math teacher's name

Solutions and Results: The solutions to the problems in this competition will be posted on the Department of Mathematics, Fresno State's website once the competition is closed. Students who score at least 50% will have their names, school affiliation, and grade level posted on our department's website. We will also have an award ceremony for the top 3 solutions sometime in Spring 2016 and winners will be notified of this event.

1. In 1961, the British theoretical physicist M. E. Fisher solved a famous and very tough problem. He showed that an 8×8 chessboard can be covered by 32 dominoes (2×1 tiles) in 12,988,816 ways. Now, one corner of the chess board is cut off and moved to a place adjacent to the opposite corner (see image below). How many ways are there to cover the 64 squares of the mutilated chessboard with 32 dominoes?



2. What is the coefficient of x^{11} in the expansion of $(1 + x^2)^4(1 + x^3)^7(1 + x^4)^{12}$?
3. Find the area of a quadrilateral $ABCD$ with $AB = 1$, $BC = 2$, $CD = 3$, $DA = 4$, and $\angle ABC = 90^\circ$.
4. Box 1 contains three cards numbered 1, 2, and 3. Box 2 contains five cards numbered 1, 2, 3, 4, and 5. Box 3 contains seven cards numbered 1, 2, 3, 4, 5, 6, and 7. A card is drawn from each of the boxes. Let x_i be the number on the card drawn from the i^{th} box, $i = 1, 2, 3$.
- (a) What is the probability that $x_1 + x_2 + x_3$ is odd?
- (b) What is the probability that x_1, x_2, x_3 are in arithmetic progression?
5. Find all possible positive integral values of a and b for which there exists a real number c such that the roots of the equation $x^3 - 3ax^2 + bx + 18c = 0$ form a non-constant arithmetic progression and the roots of the equation $x^3 + bx^2 + x - c^3 = 0$ form a non-constant geometric progression.