

Preview Problems

Grades 8-10 (in AY 2017-18)

Name: _____

- The first five problems are from a problem solving contest held as part of the Fresno Math Circle in May 2017. We frequently do problems like these in our meetings. In addition, we solve puzzles, play math strategy games, and do various group activities. Some examples are given as problems 6 and 7 here.
- Spend as much time as needed on these problems. Do not worry if you do not solve all problems. These problems are challenging. While a number of our participants solved a few of them, none solved all problems. These problems are meant for you to see if you enjoy the problems we do at the Fresno Math Circle.
- For each problem, explain how you solved it (and show your calculations), and write your answer in the answer box. Please provide good and clear explanations in full sentences. We would like to see your reasoning, not just a correct answer.
- Have fun! If you enjoy solving problems and puzzles like these, you will definitely enjoy participating in the Fresno Math Circle.
- Please scan your solutions and send them to fresnomathcircle@gmail.com no later than 1 week after the application date. Your work will be reviewed along with the application form.

1. A woman drives from her home at 30 miles per hour to the hardware store, which is 20 miles from her home. On the return trip, she encounters heavy traffic and averages 12 miles per hour. How much time does she take in driving to and from the hardware store?

Answer:

2. If I start with 2 and count by 3's until I reach 449, I will get 2, 5, 8, 11, ..., where 2 is the first number, 5 is the second number, 8 is the third number, and so on. If 449 is the N -th number, what is the value of N ?

Answer:

3. If you multiplied thirty-five 3's together (i.e. $3 \times 3 \times 3 \times \dots \times 3$, with thirty-five 3's), what would be the ones digit of the product?

Answer:

4. Let D be the sum of the odd numbers from 1 through 99 inclusive, and let N be the sum of the even numbers from 2 through 98 inclusive:

$$D = 1 + 3 + 5 + \cdots + 99,$$

$$N = 2 + 4 + 6 + \cdots + 98.$$

Which is greater, D or N ?

Answer:

5. Suppose the average of 15 consecutive numbers is 15. What is the average of the first five numbers of the set?

Answer:

6. In each puzzle, each letter represents a digit. If a letter appears more than once in one puzzle, then it represents the same digit in all of those instances. However, different letters represent different digits. Determine the value of each letter.

$$\begin{array}{r} F \quad F \\ U \quad U \\ + \quad N \quad N \\ \hline F \quad U \quad N \end{array}$$

7. This game is called Game 24. The goal is to make the quantity 24 using each of the following numbers exactly once and any operations and parentheses, in as many different ways as possible:

$$\frac{1}{2} \quad 2 \quad 3 \quad 6.$$

For example, here is one way: $6 \times (3 + \frac{1}{2} \times 2)$.

Can you think of some other ways to make 24 using these numbers?