

Fresno Math Circle

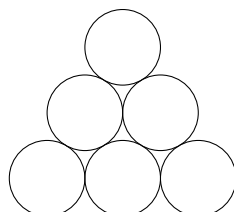
Preview Problems

Grades 9-12

Name: _____

- The first five problems similar to those we solve in our Math Circle meetings. The last two are from a Problem Solving Contest for high school students held at Fresno State.
- Spend as much time as needed on these problems. Do not worry if you do not solve all problems. These problems are very challenging and are meant for you to see if you enjoy the problems we do at the Fresno Math Circle. However, please do try your best.
- For each problem, explain how you solved it (and show your calculations), and write your answer in the answer box. Please provide good and clear explanations in full sentences. We would like to see your reasoning, not just a correct answer.
- Have fun! If you enjoy solving problems and puzzles like these, you will definitely enjoy participating in the Fresno Math Circle.
- Please scan your solutions and send them to fresnomathcircle@gmail.com no later than one week after the application date. Your work will be reviewed along with the application form.

1. In the following figure, three disks are to be colored blue, 2 are to be colored red, and 1 is to be colored green. Diagrams that can be obtained from one another by a rotation or a reflection of the whole page are considered the same. How many different diagrams are possible?



Answer:

2. Solve:

$$\begin{cases} \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{10}{3} \\ x + y = 10 \end{cases}$$

Answer:

3. Two hikers started at dawn, one from A to B and the other from B to A . Each maintained his own constant speed for the whole hike. They met at noon (they took the same trail), and continued each his way. The first one arrived at B at 4PM and the other one arrived at A at 9PM. What time was dawn that morning?

Answer:

4. When all like terms in the expression

$$(x + y + z)^{2017} + (x - y - z)^{2017}$$

are collected, how many different terms does the expression have in it?

Answer:

5. In a triangle ABC , $AB = AC$, AD is the median, E is the foot of perpendicular drawn through D to AC , and F is the midpoint of DE . Prove that AF is perpendicular to BE .

Answer:

6. A rising number, such as 34689, is a positive integer each digit of which is larger than each of the digits to its left. When all five-digit rising numbers are arranged from smallest to largest, find the 100th number in the list.

Answer:

7. Does there exist a triangle whose heights have lengths 1 , $\sqrt{5}$, $1 + \sqrt{5}$?

Answer: