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# Analysis of surface irrigation performance terms and indices

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**Abstract**

Performance terms measure how close an irrigation event is to an ideal one. An ideal or a reference irrigation is one that can apply the right amount of water over the entire region of interest (subject region) without loss. The key phrase, used in the description of a reference irrigation, has been used as the criterion for identifying the performance terms that constitute a complete set, corresponding to any given irrigation regime. Indices have been defined to quantify the performance terms. The interrelationship between performance indices as well as the relationship between each index and the system variables have been explored. The repercussions of those observations on system design and management have been discussed. © 1997 Elsevier Science B.V.

*Keywords:* Performance; Perceived requirement; Subject region

## 1. Introduction

The ever growing demand for water among competing societal activities has put undue pressure on the resource base to the point that the need for sagacious use of water has become a global concern. Irrigation in general and surface irrigation in particular is not only a major consumer of water but also one of the most inefficient users. Progress has been made toward minimizing non-beneficial consumption of irrigation water, yet a lot remains to be done.

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In the surface irrigation literature performance is described, implicitly or explicitly, as a measure of the merit of an irrigation scenario (event); a description, though correct, not sharp enough to allow identification of a self-contained set of performance terms corresponding to any given irrigation regime. Defining performance in terms of a reference irrigation would help avoid the confusion that the above description may entail. Few authors have attempted to shed light on the meaning, related assumptions as well as methods of quantifying irrigation performance terms. The ASCE On-farm Irrigation Committee of the Irrigation and Drainage Division (1978) provided a point of departure. Bos (1979) proposed a set of equations to estimate a range of efficiency terms at different levels of system hierarchy. Blair and Smerdon (1988) presented a performance term which they called the deficit/excess efficiency. Wang et al. (1996) utilized the concept of Blair and Smerdon (1988) to define a general efficiency term in which only the transpiration component of the water taken up by plants is considered beneficially disposed. A brief, yet lucid, description of efficiency and uniformity terms along with some fundamental assumptions has been presented by Clemmens et al. (1995). Clemmens (1995) proposed a method for assessing the accuracy of irrigation efficiency estimates. Burt et al. (1995) summarized ways of identifying and quantifying irrigation efficiency and uniformity components. Clemmens (1988) and Clemmens and Solomon (1995) proposed a stochastic approach for combining the different components that contribute to distribution uniformity.

In this article an attempt is made to provide satisfactory answers to two important questions: (1) what does the term performance mean, and (2) what constitutes a complete (self-contained) set of performance terms under any given irrigation regime? Moreover, the paper presents descriptions of the surface irrigation performance indices and detailed procedures for their quantitative evaluation. The interrelationship between performance indices as well as the relationship between performance indices and the system variables have been investigated. Other supplementary indices of design and management importance, i.e. deep percolation fraction and runoff fraction, have been defined and their influence on design and management decisions have been discussed.

## 2. Assumptions and definition of terms

A reference (an ideal) irrigation event (operation scenario) is one that can apply the right amount of water over the entire region of interest (subject region) without loss. In practice, however, excess and uneven application of irrigation water are the 'twin facts of life' that irrigators ought to live with. Irrigators cannot do without them but ought to strive to minimize them. An ideal irrigation can therefore be used as a reference to assess the merit of a real life irrigation event. It then follows that performance can be defined as a measure of "how close an irrigation event (scenario) is to the reference irrigation".

Evident from the above description of performance is that it has to be quantified in terms of a *perceived requirement*, which in turn needs the definition of a *subject region*. A subject region can be defined as a region from which crops extract water to meet their consumptive use demand. The surface area of a subject region can loosely be defined as the area of the irrigated field on *plan*. Nonetheless, with the view of accounting for the limitations of available performance evaluation tools (e.g. mathematical models), it is defined in a more restrictive sense as one whose areal extent on *plan* is that of the area of a channel of unit characteristics breadth (i.e. a furrow or a unit width basin or a unit width border as the case may be). A simplification concomitant to the forgoing description of a subject region is that across the direction of irrigation, performance is assumed uniform. Vertically, the bottom of the effective root depth coincides with the bottom boundary of the subject region, while the soil surface serves as its top boundary.

The following assumptions have been made: (1) Only that fraction of the applied water that has infiltrated into and been retained in the subject region is considered as beneficially disposed. The water that leaves the subject region through its boundaries is considered unavailable for plant use and as such categorized as a loss. Observe that this definition excludes the leaching requirement from the domain of beneficially disposed water. Moreover, evaporation from the top boundary as well as contribution to the subject region from groundwater sources during an irrigation event are assumed to be of negligible quantity. (2) Soil is homogeneous and a single crop is grown over the subject region and the influence of micro-topography on the unevenness of irrigation water application is assumed to be marginal. The net effect of the latter assumptions is that water requirement is spatially uniform and that uneven application of water over the subject region is entirely due to spatial variation in intake opportunity time. (3) There exists a critical moisture content, of the soil of the subject region, a fall below which may trigger unacceptable levels of yield reduction. A *perceived requirement* can therefore be defined as the amount of water that the soil of the subject region takes into storage as its moisture content is increased from the critical value to field capacity (FC). Implicitly assumed in this description of adequacy is that in order to sustain normal crop growth and obtain a satisfactory yield, the *perceived requirement* must be closely satisfied in every event.

Fig. 1 depicts a typical irrigation water destination diagram. R1 is water stored in the subject region and is considered as beneficially disposed, R2 is the channel reach in which irrigation falls short of the perceived requirement, and R3 is the excess water which percolated below the bottom boundary of the subject region. Figs. 1(a) and (c) depict the inflow and runoff hydrographs. Depending on the relationship between the net irrigation requirement ($Z_r$), the maximum infiltrated amount ($Z_{max}$), and the minimum

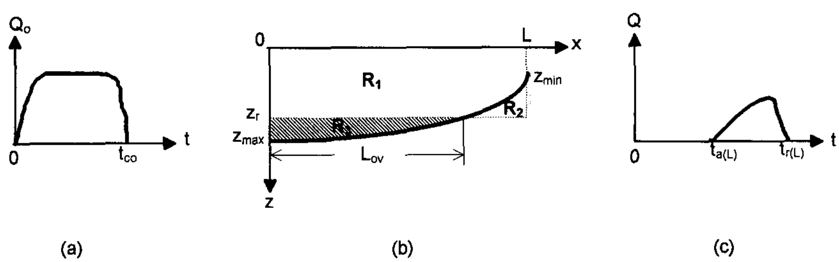


Fig. 1. Schematics of applied water and surface runoff in surface irrigation.

infiltrated amount ($Z_{min}$), an irrigation event can fall into one of the following three regimes:

$$\text{Regime } 1 = Z_{max} \leq Z_r, \quad \text{Regime } 2 = Z_{min} < Z_r < Z_{max}, \quad \text{Regime } 3 = Z_r \leq Z_{min} \tag{1}$$

## 3. Performance terms

Two key phrases have been used in the description of a reference irrigation: (1) "without loss" and (2) the "right amount over the entire subject region". These phrases would be used to identify the performance terms that constitute a complete set, corresponding to the three irrigation regimes described above.

1. Excess application of irrigation water, though unavoidable in a real life situation, must be minimized (*without loss ~ minimum loss*). Application efficiency ($E_a$) is the index which is used as a measure of how effective an irrigation event is in minimizing unavoidable losses. Observe that application efficiency provides only a partial view of the whole. It supplies no answer what so ever to the question of how closely the perceived requirement (the *right amount*) is satisfied *over the entire subject region*, thus the need for a separate index.
2. Adequate irrigation, evaluated in terms of the perceived requirement (the *right amount*), over the entire subject region is necessary to sustain normal crop growth and obtain a satisfactory yield. Water requirement efficiency ($E_r$) is used as a measure of how close the applied amount is to the perceived requirement (the *right amount*) over the entire subject region. Although adequacy is a spatially variable term, due to unavoidable spatial variation in intake opportunity time, a mean value of $E_r$ is used to describe it. It is in response to the need to complement this inherent weakness of the $E_r$ index, of being unable to account for the spatial variability of the adequacy term, that a third criterion has been introduced.
3. The spatial uniformity (evenness) of irrigation water application provides a vital clue as to how good the corresponding $E_r$ index is as a representative measure of adequacy over the entire subject region. Distribution uniformity (DU) and Christiansen's uniformity coefficient (UCC) are the most commonly used uniformity indices in surface irrigation applications.

It is interesting to note that when the subject region is under irrigated over its entire length or partly under irrigated, the uniformity indices complement the information, that the $E_r$ index provides, on adequacy. That this, indeed, is the case has been demonstrated at a later point in the paper by analyzing the equations that relate the $E_r$ index with the uniformity indices. On the other hand, when the subject region receives adequate irrigation over its entire length, $E_r = 100\%$ and adequacy is no more a spatially variable term. Therefore, the question of how close the applied amount to the right amount over the entire subject region can be answered by the $E_r$ index alone. In such a situation, the uniformity indices are redundant as performance criteria. It is true that the uniformity indices can still tell us how water is distributed along the length of run of the channel and from that we can obtain qualitative information as to how the irrigation water loss is divided into its components. More specific information on this can, however, be secured by calculating the components of irrigation water loss directly. Besides, this information is needed not as a measure of performance ("how close the current event (scenario) is to the reference or ideal one") as such, but as an answer to the question of how to improve the performance of an irrigation scenario (a design or management problem). Stated in other words, what the above means is that it is true that the uniformity indices possess information that the $E_a$ and $E_r$ indices do not, but that the information as such is of no use in evaluating performance as defined herein. It then follows that (1) in a completely over irrigated situation, $E_a$ and $E_r$ alone can fully define the performance of a system. In fact, if the target $E_r$ is set to 100%, it then remains to determine the maximum $E_a$ to identify the irrigation scenario with the best performance; (2) there exists no irrigation regime which requires evaluation of the total irrigation water loss or its individual components to asses performance. However, the individual terms and corresponding indices that are used to quantify irrigation water loss are important in their own right. They provide vital information that would help guide design and management decisions and are therefore described in the sequel.

## 4. Terms of design and management importance

There are two terms of relevance in this category: (1) Runoff loss which represents that part of the applied water that has left the subject region as surface runoff. Runoff fraction, $R_f$, is the index used to quantify the runoff loss. (2) Deep percolation loss which represents that portion of the irrigation water loss that is attributed to percolation below the bottom boundary of the subject region. Deep percolation fraction, $D_f$, is the index used to quantify the deep percolation loss.

## 5. Definition of performance indices

### 5.1. Application efficiency ($E_a$)

Application efficiency can be defined as the ratio of the volume of water stored in the subject region to the volume of water diverted into the subject region (Fig. 1). A general expression for $E_a$ is:

$$E_a = \frac{\int_0^L Z \mathrm{d}x - \int_0^{L_{ov}} Z \mathrm{d}x + Z_r L_{ov}}{\int_0^{t_{co}} Q_o \mathrm{d}t} 100 \tag{2}$$

where $Z$ = cumulative infiltration expressed as a function of distance ($m^3$ $m^{-1}$), $L$ = channel length (m), $L_{ov}$ = the length over which the infiltrated amount equals or exceeds the requirement (m), $Z_r$ = net irrigation requirement or the perceived requirement ($m^3\ m^{-1}$). The first term, in the numerator, represents the volume of water infiltrated over the entire reach of the subject region. The second term of the numerator stands for the total volume of water infiltrated over that part of the channel which receives irrigation amounts at least equal to the perceived requirement, $Z_r$. The third term, in the numerator, represents the volume of water retained in the subject region over the reach that receives excess irrigation. The expression in the denominator represents the total volume admitted into the subject region. In practice inlet flow rate does not vary continuously with time, but rather in a step-wise fashion. Exact expression can, therefore, be proposed for the denominator of Eq. (2) as follows:

$$\int_0^{t_{co}} Q_o \mathrm{d}t = \Sigma_{i=1}^{I} Q_{oi} \Delta t_i \tag{3}$$

where $i$ = time index, $I$ = the total number of time intervals, $\Delta t_i$ = the $i$th time interval during which the inlet flow rate is set at $Q_{oi}$, $Q_{oi}$ = inlet flow rate during time period $\Delta t_i$. Exact solutions to the integral expressions in the numerator of Eq. (2) are possible only if advance and recession times can be expressed as explicit distance dependent functions. In general it is convenient and in the case of furrows, probably, more accurate to use methods of quadrature to evaluate the integral expressions in the numerator of Eq. (2). Both scenarios are explored in the sequel.

### 5.1.1. Evaluation of the $E_a$ equation

Depending on the prevailing irrigation regime, any irrigation event can be classified into one of the following three particular cases.

Regime 1 ($Z_r \leq Z_{min}$): this is a situation in which the length of over irrigation, $L_{ov}$, is equal to the length of run of the channel. Eq. (2) therefore reduces to:

$$E_a = \frac{Z_r L}{\Sigma_{i=1}^{I} Q_{oi} \Delta t_i} 100 \tag{4}$$

Regime 2 ($Z_{min} < Z_r < Z_{max}$): this is a case in which all three terms in the numerator of Eq. (2) have to be evaluated, either using direct analytical or using numerical integration.

1. The quadrature (numerical integration) approach: an array of methods does exist. Using Simpson's rule, for example, the first term in the numerator of Eq. (2) can be expressed as:

$$\int_0^L Z \mathrm{d}x = \frac{\Delta L}{3}[Z_0 + 4Z_1 + 2Z_2 + 4Z_3 + 2Z_4 + \ldots + 4Z_{N-1} + Z_N] \tag{5}$$

   where $\Delta L$ = constant distance interval between computational nodes or observation stations ($\Delta L = L/N$); $N$ = number of intervals (must be even number); $Z_i$ = infiltrated amount at station $i$ ($m^3\ m^{-1}$). The second term in the numerator of Eq. (2) can also be evaluated using Eq. (5), only by changing the upper limit of integration from $L$ to $L_{ov}$. Note that the distance, from the upstream end, of a station with a $Z_i$ value nearest to $Z_r$ can be taken as an approximate value of $L_{ov}$.
2. Analytical approach: involves direct integration of terms in Eq. (2). Eq. (6) is an

approximate analytical expression for the first term in the numerator of Eq. (2), using the modified Kostiakov infiltration function.

$$\int_0^L Z\mathrm{d}x = k\int_0^L \sum_{m=0}^{M} \binom{a}{m} (-t_\mathrm{a}(x))^m t_\mathrm{r}(x)^{a-m}_{\mathrm{d}x} - c\int_0^L (t_\mathrm{r}(x) - t_\mathrm{a}(x))\mathrm{d}x \quad (6)$$

where $x$ = distance measured from the upstream end of the channel (m), $t_\mathrm{r}(x)$ and $t_\mathrm{a}(x)$ = explicit distance dependent functions for recession time and advance time, respectively (min); $k$ (m$^3$ m$^{-1}$ min$^{-a}$), $a$ (dimensionless), and $c$ (m$^3$ m$^{-1}$ min$^{-1}$) = respectively, coefficient of the power term, exponent of the power term, and coefficient of the linear term of the modified Kostiakov infiltration function; $m = 0, 1, 2, 3, \ldots, M$; and $M$ = an integer, the value of which has to be selected on the basis of the desired accuracy. Observe that Eq. (6) can be used to evaluate the second term in the numerator of the right-hand side of Eq. (2), only by changing the upper limit of integration from $L$ to $L_{\mathrm{ov}}$.

The following is a procedure for estimating $L_{\mathrm{ov}}$, when the analytical approach is used to evaluate the integral expressions in Eq. (2):

1. determine the required intake opportunity time ($T_{\mathrm{req}}$) (that is the time required to infiltrate $Z_\mathrm{r}$);
2. make an initial estimate of $L_{\mathrm{ov}}^{(1)}$ (a reasonable initial estimate would be $L_{\mathrm{ov}}^{(1)} = L/2$);
3. use Eq. (7), which is basically the recursive formula for a 1-D Newton–Raphson iteration, to estimate $L_{\mathrm{ov}}^{(i+1)}$ from $L_{\mathrm{ov}}^{(i)}$.

$$L_{\mathrm{ov}}^{(i+1)} = L_{\mathrm{ov}}^{(i)} - \frac{T_{\mathrm{req}} - t_\mathrm{r}\left(L_{\mathrm{ov}}^{(i)}\right) + t_\mathrm{a}\left(L_{\mathrm{ov}}^{(i)}\right)}{\left.\dfrac{\mathrm{d}t_\mathrm{a}(x)}{\mathrm{d}x}\right|_{x=L_{\mathrm{ov}}^{(i)}} - \left.\dfrac{\mathrm{d}t_\mathrm{r}(x)}{\mathrm{d}x}\right|_{x=L_{\mathrm{ov}}^{(i)}}} \quad (7)$$

where $i$ = iteration counter;

4. stop iteration when a certain prescribed error tolerance criterion/ia is/are satisfied. The error can be defined in terms of $L$ as $\Delta_L = |L_{\mathrm{ov}}^{(i+1)} - L_{\mathrm{ov}}^{(i)}|$ and/or in terms of time as $\Delta_t = |T_{\mathrm{req}} - t_\mathrm{r}(L_{\mathrm{ov}}^{(i+1)}) + t_\mathrm{a}(L_{\mathrm{ov}}^{(i+1)})|$.

Regime 3 ($Z_{\max} \le Z_\mathrm{r}$): in this case the second and third terms of the numerator are zero, thus Eq. (2) becomes:

$$E_\mathrm{a} = \frac{\int_0^L Z\mathrm{d}x}{\sum_{i=1}^{I} Q_{\mathrm{o}i}\Delta t_i} 100 \quad (8)$$

The numerator of Eq. (8) can be evaluated using Eq. (5) or Eq. (6), as the case may be.

### 5.2. Water requirement (storage) efficiency ($\mathrm{E}_r$)

Water requirement efficiency can be defined as the ratio of the volume of water actually stored in the subject region to the volume of water that can be stored (Fig. 1).

The general form of the $E_r$ equation is:

$$E_r = \frac{\int_0^L Z \mathrm{d}x - \int_0^{L_{ov}} Z \mathrm{d}x + Z_r L_{ov}}{Z_r L} 100 \tag{9}$$

### 5.2.1. Evaluation of the $E_r$ equation

Depending on the prevailing irrigation regime, the following three particular solutions of Eq. (9) can be discerned:

Regime 1 ($Z_r \leq Z_{min}$): this implies that $L = L_{ov}$, thus $E_r = 100\%$.

Regime 2 ($Z_{min} < Z_r < Z_{max}$): this is the general case, in which Eq. (9) has to be considered in its complete form. Eq. (5) or Eq. (6) is to be used in evaluating the integral expressions in the numerator of Eq. (9).

Regime 3 ($Z_{max} \leq Z_r$): in this case Eq. (8) can be reduced to the following form:

$$E_r = \frac{\int_0^L Z \mathrm{d}x}{Z_r L} 100 \tag{10}$$

Eq. (5) or Eq. (6) can be used to estimate the integral expression in the numerator of Eq. (10).

## 5.3. Irrigation uniformity

The third remaining performance term is uniformity. Two of the most commonly used uniformity indices, in surface irrigation applications, are: distribution uniformity (DU) and Christiansen's uniformity coefficient (UCC).

### 5.3.1. Distribution uniformity

DU is defined as the ratio of the minimum infiltrated amount to the average infiltrated amount over the entire length of the channel. A general expression for DU is:

$$\mathrm{DU} = \frac{Z_{min}}{Z_{av}} 100 \tag{11}$$

where $Z_{av}$ = average infiltrated amount over the length of run of the channel ($m^3\ m^{-1}$). $Z_{av}$ can be expressed as follows:

$$Z_{av} = \frac{\int_0^L Z \mathrm{d}x}{L} \tag{12}$$

The numerator of Eq. (12) can be approximated using either Eq. (5) or Eq. (6), as the case may be.

*5.3.2. Christiansen's uniformity coefficient*

UCC is defined as the ratio of the difference between the average infiltrated amount and the average deviation from the average infiltrated amount to the average infiltrated amount. The UCC equation is:

$$\mathrm{UCC} = \left[1 - \frac{\sum_{i=1}^{N} |Z_i - Z_{\mathrm{av}}|}{Z_{\mathrm{av}} N}\right] 100 \tag{13}$$

where $Z_i$ = infiltrated amount at point $i$ ($\mathrm{m}^3\ \mathrm{m}^{-1}$), $N$ = number of points used in the computation of UCC.

## 6. Irrigation water loss indicators

### 6.1. Deep percolation fraction ($D_f$)

The deep percolation fraction is defined as the ratio of the volume of water percolated below the bottom boundary of the subject region to the total volume admitted into the subject region. The following is a general form of the $D_{\mathrm{f}}$ equation:

$$D_{\mathrm{f}} = \frac{\int_0^{L_{\mathrm{ov}}} Z \mathrm{d}x - Z_{\mathrm{r}} L_{\mathrm{ov}}}{\sum_{i=0}^{I} Q_{\mathrm{o}i} \Delta t_i} 100 \tag{14}$$

Depending on the prevailing irrigation regime, the following particular cases can be discerned in solving Eq. (14):

Regime 1 ($Z_{\mathrm{r}} \le Z_{\mathrm{min}}$): this is a case in which $L_{\mathrm{ov}} = L$. The integral expression in the numerator can be solved using the procedure outlined in connection with Eq. (2).

Regime 2 ($Z_{\mathrm{min}} < Z_{\mathrm{r}} < Z_{\mathrm{max}}$): Eq. (14) is directly applicable. The integral expression in the numerator can be evaluated using the procedure described in connection with Eq. (2).

Regime 3 ($Z_{\mathrm{max}} \le Z_{\mathrm{r}}$): this implies that $L_{\mathrm{ov}} = 0$, therefore $D_{\mathrm{f}} = 0$.

### 6.2. Runoff fraction ($R_f$)

The runoff fraction can be defined as the ratio of the volume of runoff to the volume of water diverted into the subject region. Eq. (15) is the general expression for $R_{\mathrm{f}}$:

$$R_{\mathrm{f}} = \frac{\int_{t_{\mathrm{a}}(L)}^{t_{\mathrm{r}}(L)} Q_L(t) \mathrm{d}t}{\sum_{i=1}^{I} Q_{\mathrm{o}i} \Delta t_i} 100 \tag{15}$$

where $t_a(L)$ and $t_r(L)$ = advance time and recession time corresponding to the downstream end of the channel (m), and $Q_L(t)$ = time dependent runoff rate function at the downstream end ($m^3$ $min^{-1}$). Generally direct analytical integration of Eq. (15) is not possible. The reason is that there does not exist an explicit time dependent equation, $Q_L(t)$, describing the runoff hydrograph. Therefore, the methods of quadrature are commonly used to approximate the surface runoff volume. This is true for all surface irrigation models except the volume balance model, in which case the dynamic equation that describes flow variation in time and space is entirely neglected and hence there is no way that one can determine even numerically the integral expression in the numerator of Eq. (15). In such cases a simple mass balance approach, Eq. (16), can be used to estimate $R_f$.

$$R_f = 100 - E_a - D_f \tag{16}$$

## 7. The interrelationship of performance indices

### 7.1. Application efficiency and water requirement efficiency

Using Eqs. (2) and (9), one can show that $E_a$ and $E_r$ can be related as follows:

$$E_a = E_r \frac{Z_r L}{\int_0^{t_{co}} Q_o \mathrm{d}t} = E_r \frac{Z_r L}{\sum_{i=1}^{I} Q_{oi} \Delta t_i} \tag{17}$$

This equation can be used to calculate $E_a$ from $E_r$ and vice-versa, but is most useful in exploring the interdependency of the two indices. The following observations can be made of Eq. (17): (1) Given the ratio of the volume of water needed to refill the storage capacity of a subject region to the total volume of water available to irrigate a subject region, $E_a$ is a linear increasing function of $E_r$. (2) For a given $L$, $Z_r$, and target $E_r$ combination $E_a$ is a function of the volume of water admitted into the subject region alone. In such a situation, if a target $E_a$ value is chosen a priori, the corresponding $Q_o$–$t_{co}$ combination can be selected such that the condition imposed on the target $E_r$ value is satisfied and that the product of $Q_o$ and $t_{co}$ equals the total volume available to irrigate the subject region, which is given as $E_r^* Z_r^* L/E_a$. (3) The relationship between optimum $E_a$ and the system variables, given a target level of $E_r$, can be investigated and a general rule for identifying the optimum $E_a$ as a function of the system variables can be developed. A discussion on this is in order at a later point.

### 7.2. Water requirement efficiency and distribution uniformity

Combining Eqs. (10)–(12) would result in the following expressions for DU in terms of $E_r$, $Z_{min}$, and $Z_r$.

$$\mathrm{DU} = \frac{\left(\dfrac{Z_{min}}{Z_r}\right)}{\left(\dfrac{E_r}{100}\right)} 100 \Leftrightarrow \mathrm{DU}^* = \frac{Z_{min}^*}{E_r^*} \tag{18}$$

where $DU^* = DU$ expressed as a fraction, $Z^*_{min}$ = minimum infiltrated amount normalized at $Z = Z_r$, $E_r^* = E_r$ expressed as a fraction. One can observe that for the type of irrigation regime considered herein (i.e. $Z_{max} \le Z_r$), $E_r^*$ is equal to the average infiltrated amount normalized at $Z = Z_r$ ($Z^*_{av}$). Moreover, too low $DU^*$ is associated with a situation in which $Z^*_{min}$ is significantly smaller than $Z^*_{av}$ ($E_r^*$), in which case both $Z^*_{av}$ and $E_r^*$ are biased towards the higher values of Z, hence not a measure representative of the adequacy of irrigation over the entire reach of the subject region. Stated differently, what this means is that when the $E_r$ index is not a representative measure of adequacy over the entire subject region, the situation is highlighted by a low value of DU. In what follows a similar analysis is presented for $E_r$ versus UCC.

### 7.3. Water requirement efficiency and Christiansen's uniformity coefficient

The following expression can be obtained for UCC in terms of $E_r$, $Z_r$, and $Z_i$ using Eqs. (10), (12) and (13), when $Z_{max} \le Z_r$:

$$\mathrm{UCC} = \left[1 - \frac{\sum_{i=1}^{N} |Z_i - \frac{E_r}{100} Z_r|}{\frac{E_r}{100} Z_r N}\right] 100 \Leftrightarrow \mathrm{UCC}^* = \left[1 - \frac{1}{N}\sum_{i=1}^{N} \left|\frac{Z_i^*}{E_r^*} - 1\right|\right] \qquad (19)$$

where $UCC^* = UCC$ expressed as a fraction, $E_r^* = E_r$ expressed as a fraction, $Z_i^*$ = infiltrated amount at node $i$ normalized at $Z = Z_r$. Note that for the irrigation regime under consideration (i.e. $Z_{max} \le Z_r$), $E_r^*$ is equal to the average infiltrated amount normalized at $Z = Z_r$ ($Z^*_{av}$). Eq. (19) shows that a too low value of UCC is associated with a situation in which the values of $Z_i^*/E_r^*$ (or $Z_i^*/Z^*_{av}$) are significantly smaller or significantly larger than unity for a substantial number of the computational nodes (stations). This in turn means that both $Z^*_{av}$ and $E_r^*$ are biased towards the upper or the lower limit of the range of the observed $Z_i^*$ s. What this tells us is that, when the water requirement efficiency is not a representative measure of the adequacy of irrigation over the length of run of the subject region, the situation is highlighted by a low value of UCC.

The above discussion on the relationships between $E_r$ and DU as well as $E_r$ and UCC is an attempt to explain the function that the uniformity indices serve in complementing the information provided by the $E_r$ index, when the condition of $Z_{max} \le Z_r$ is satisfied. It is expected that this relationship holds true for the irrigation regime in which $Z_{min} < Z_r < Z_{max}$, although the form of the $E_r$ equation associated with this irrigation regime is such that it does not lend itself to the kind of simple analysis presented for the case in which $Z_{max} \le Z_r$.

### 7.4. Application efficiency and uniformity

Generally both DU and UCC can be defined in terms of $E_a$. However, the resulting relationships are not as valuable as those obtained for $E_r$ versus DU and Er versus UCC. In addition, one may observe that Eqs. (4) and (11) are equivalent expressions when

$Z_r = Z_{min}$ and $R_f = 0$ (e.g. in a basin). What this, in other words, means is that in basins, when $Z_r = Z_{min}$, $E_a = DU$. In such a situation, $E_r = 100\%$. Thus, there is no need to quantify uniformity. It therefore remains to determine $E_a$ only to evaluate the performance of an irrigation event.

## 8. Relationships between performance/related indices and system variables ($Q_o$ and $L$)

For a given $Z_r$ and field condition, adequacy of applied water is a primary criterion used in selecting an appropriate value of cutoff time, $t_{co}$. The optimum $E_r$ level to sustain normal crop growth is known and it is 100% (of course, this is on the premise that all other inputs, needed for normal plant growth, are set at their optimal level). It is thus a simple matter to estimate required cutoff time, $t_{co}$, given $Z_r$ and an allowable deficit for any given irrigation situation. It is, however, important to note that this should by no means imply that $t_{co}$ is unrelated to other system parameters and variables. What it simply means is that given a certain system parameter and variable levels combination, the desired level of adequacy determines the specific value that $t_{co}$ may assume.

Given a target $E_r$, the task of selecting the combination of inlet flow rate, $Q_o$, and channel length, $L$, that would result in optimum application efficiency, $E_a$, and acceptable level of uniformity, is not an easy one. It requires understanding of how performance indices are related to these variables. Observations made on the responses of the system performance and related indices to simultaneous changes in both system variables ($Q_o$ and $L$) would be desirable and more informative in establishing those relationships. Such a study, however, is too complex and hence beyond the scope of this study. Instead, the effect of changes on $Q_o$ and $L$ on the performance indices has been investigated on a one at a time basis, for a situation in which $E_r = 100\%$ and $E_r = 90\%$, and the results have been presented in Figs. 2–9. With respect to the relationship between the performance indices and the system variables ($Q_o$ or $L$), two major patterns can be discerned, i.e. one for basins and another for borders and furrows. This difference can be attributed to differences in bed slope and downstream boundary conditions between basins on the one hand and borders and furrows on the other.

### 8.1. Graded free-draining channels

The FURrow Design-management and EValuation software package (FURDEV, Zerihun and Feyen, 1996) has been used in this analysis. The response of system performance indices to variations in $Q_o$ and $L$ are presented in Figs. 2 and 3, for the case in which $E_r = 100\%$, and in Figs. 4 and 5, for the case in which $E_r = 90\%$. The following must be observed in interpreting the curves presented here: (1) the results presented are site and irrigation specific, (2) the analysis has been made only for two values of $E_r$ (Table 1). The fact that the second requirement does not pose significant restriction on the general applicability of the curves is readily apparent, for in practice systems are designed and operated to apply near adequate amounts over the entire reach of a subject region. In addition, intuitive reasoning as well as observations made on

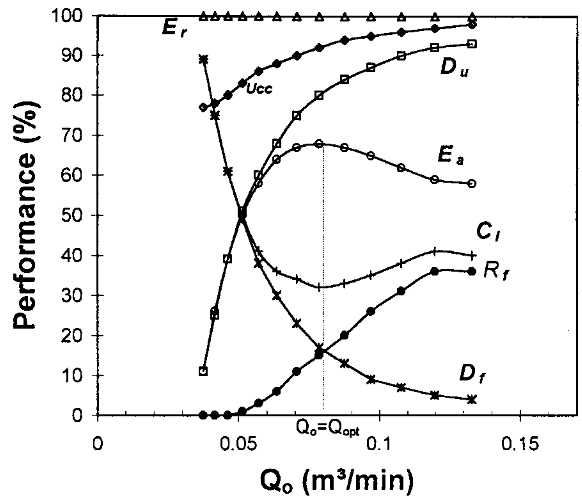


Fig. 2. Performance indices versus inlet flow rate, for a free-draining graded channel ($E_r$ = 100%).

several simulation results, corresponding to different parameter sets, point to the fact that the trends observed in the cases presented herein (Table 1) may generally hold true, though slight variations in curvature and slope of the performance curves are unavoidable as the field condition changes. Thus, the first restriction too may not pose a serious impediment to the general applicability of the deductions that stem from the current analysis.

It can be noted from Figs. 2 and 6 that as inlet flow rate, $Q_o$, is increased steadily from a minimum value, $Q_{min}$, (i.e. a value sufficient to advance to the downstream end

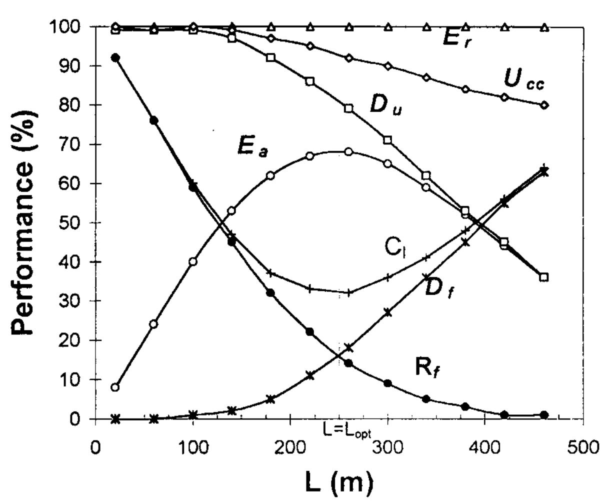


Fig. 3. Performance versus length of run, for a free-draining graded channel ($E_r$ = 100%).

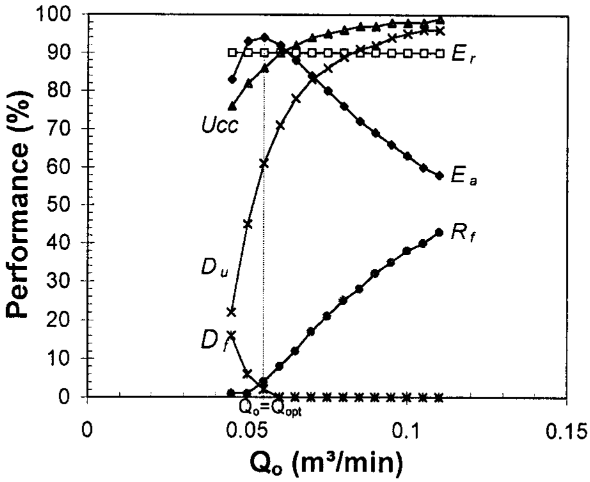


Fig. 4. Performance indices versus inlet flow rate, for a free-draining graded channel ($E_r = 90\%$).

of the channel) to a maximum value, $Q_{max}$, (say the maximum non-erosive flow rate) the deep percolation fraction, $D_f$, continually declines. On the other hand, a steady increase of the inlet flow rate results in an increase in the runoff fraction, $R_f$. The combined irrigation water loss, $C_l$, which represents the sum of $D_f$ and $R_f$ decreases as $Q_o$ increases from $Q_{min}$ to a value which, for now, we simply refer to as $Q_{opt}$, and then starts to increase as $Q_o$ is increased beyond $Q_{opt}$. The change in the algebraic sign of the slope of the $C_l$ curve occurs at about $Q_o = Q_{opt}$. Observe that at this point $D_f \approx R_f$ and that $Q_{opt}$ corresponds to the minimum combined surface runoff and deep percolation

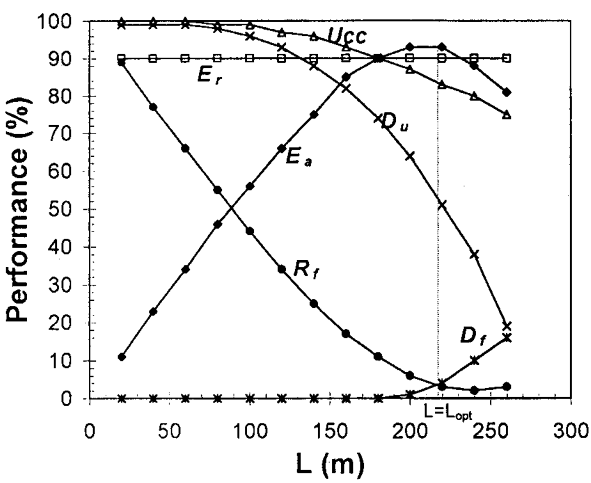


Fig. 5. Performance versus length of run, for a free-draining graded channel ($E_r = 90\%$).

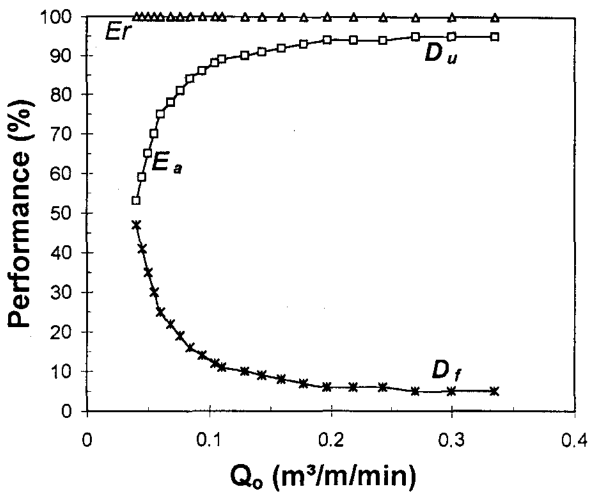


Fig. 6. Performance indices versus inlet flow rate, for a basin ($E_r = 100\%$).

losses. Applying the law of conservation of mass $E_a$ can be expressed as 100 less $C_l$. The resulting $E_a$ curve can then be described as, first, an increasing function of $Q_o$ which attains its maximum value when $C_l$ is minimum and $Q_o = Q_{opt}$ and then declines as $Q_o$ is further increased. Observe that $Q_{opt}$ is the optimal flow rate with respect to $E_a$. The following important inferences can be made of this analysis:

1. $E_a$ and $C_l$ are unimodal functions of $Q_o$, provided all other variables remain constant. Thus, for any given parameter level combination there exists a value of $Q_o$ at which

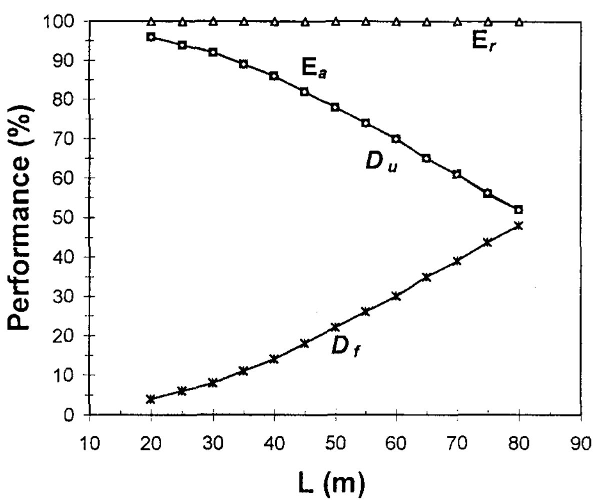


Fig. 7. Performance indices versus length of run, for a basin ($E_r = 100\%$).

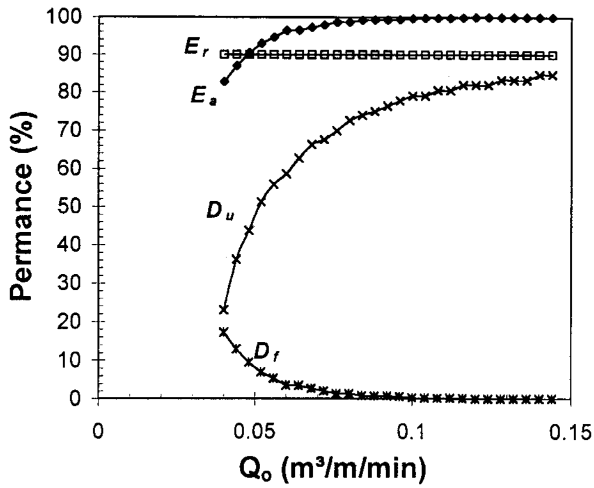


Fig. 8. Performance indices versus inlet flow rate, for a basin ($E_r = 90\%$).

efficiency is maximum; conversely $C_1$ is minimum. Four different cases can be discerned:

Case 1 ($Q_{min} \leq Q_{opt} \leq Q_{max}$): the potential maximum is within the feasible range and occurs at a $Q_o$ value which results in roughly equal $D_f$ and $R_f$.

Case 2 ($Q_{opt} \leq Q_{min}$): the potential maximum is outside the feasible range and hence the best practically attainable efficiency is the one that corresponds to $Q_o = Q_{min}$.

Case 3 ($Q_{min} \leq Q_{max} \leq Q_{opt}$): the potential maximum lies outside the feasible range, hence the best practically attainable efficiency is the one that corresponds to $Q_o = Q_{max}$.

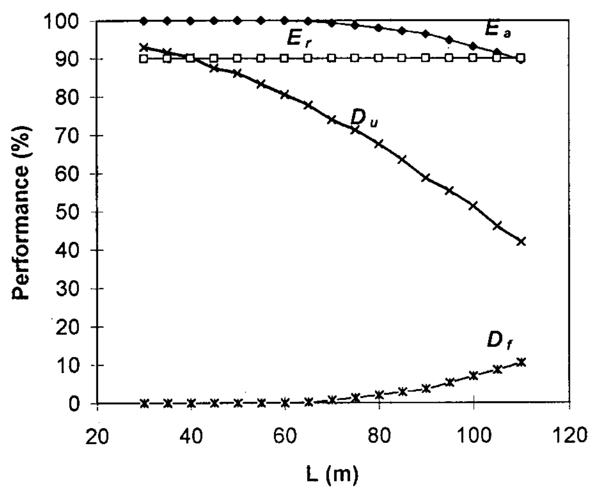


Fig. 9. Performance indices versus length of run, for a basin ($E_r = 90\%$).

Table 1
Data used in the calculation of performance indices

| List of variables and parameters | Furrow | | | | Basin | | | |
|---|---|---|---|---|---|---|---|---|
| | $E_r = 100\%$ | | $E_r = 90\%$ | | $E_r = 100\%$ | | $E_r = 90\%$ | |
| | $Q_o$ | $L$ | $Q_o$ | $L$ | $Q_o$ | $L$ | $Q_o$ | $L$ |
| $Q_o$ ($m^3$ $min^{-1}$) | – | 0.012 | – | 0.09 | – | 0.0385 | – | 0.06 |
| $L$ (m) | 200 | – | 180 | – | 80 | – | 90 | – |
| $k$ (mm $min^{-a}$) | 4.28 | 4.28 | 3.387 | 4.28 | – | – | 2.3 | 2.3 |
| $a$ (–) | 0.55 | 0.55 | 0.506 | 0.55 | – | – | 0.7 | 0.7 |
| $C_o$ (mm $min^{-1}$) | 0.2 | 0.2 | 0.213 | 0.2 | – | – | – | – |
| $\sigma_1$ ($m^{2-\sigma 2}$) | 0.987 | 0.987 | 0.987 | 0.987 | – | – | – | – |
| $\sigma_2$ (–) | 1.734 | 1.734 | 1.734 | 1.734 | – | – | – | – |
| $\gamma_1$ ($m^{1-\gamma 2}$) | 2.439 | 2.439 | 2.439 | 2.439 | – | – | – | – |
| $\gamma_2$ (–) | 0.811 | 0.811 | 0.811 | 0.811 | – | – | – | – |
| $Z_r$ (mm) | 100 | 100 | 50 | 100 | 80 | 80 | 80 | 80 |
| $n$ ($m^{1/6}$) | 0.04 | 0.04 | 0.04 | 0.04 | 0.15 | 0.15 | 0.15 | 0.15 |
| $V_{max}$ (m $min^{-1}$) | 13.8 | 13.8 | 12.1 | 13.8 | – | – | – | – |
| $S_o$ (–) | 0.008 | 0.008 | 0.006 | 0.008 | – | – | – | – |
| $I_f$ (–) | – | – | – | – | 1 | 1 | – | – |

$Q_o$ = unit flow rate; $\sigma_1$ and $\sigma_2$ represent coefficient and exponent, respectively, of the power function used to relate cross-sectional area of flow with depth of flow in the FURDEV model; $\gamma_1$ and $\gamma_2$ = coefficient and exponent, respectively, of the power function used to relate wetted perimeter and depth of flow in the FURDEV model; $n$ = Manning's roughness coefficient; $V_{max}$ = maximum non-erosive velocity; $S_o$ = bed slope; $I_f$ = SCS intake family. Note that cutoff time has been adjusted for each case, such that the criteria imposed on $E_r$ are satisfied.

Case 4 ($Q_{max} \leq Q_{min} \leq Q_{opt}$): this is a situation in which there is no solution, i.e. empty solution set.

2. As the application efficiency curve is a bell shaped unimodal curve, it is possible to operate the system at the same efficiency level by shifting from one limb of the $E_a$ curve to the other.
3. Uniformity of irrigation water application is a strictly increasing function of flow rate.
4. $D_f$ is a non-increasing function of $Q_o$.
5. $R_f$ is a non-decreasing function of $Q_o$.
6. It has been shown by Zerihun et al. (1993) that if the assumption of $E_r = 100\%$ is satisfied then the potential maximum $E_a$ is entirely dependent on system parameters.

Similar observations can be made of the relationship between performance indices and $L$ (Figs. 3 and 5). With the exception of the $E_a$–$L$ and $C_l$–$L$ curves, which are more or less of the same general shape as that of $E_a$–$Q_o$ and $C_l$–$Q_o$ curves described above, all other curves in Figs. 3 and 5 are somewhat mirror images or lateral inversions of those presented in Figs. 2 and 4. What this means is that if a performance index increases with $Q_o$ then it is a decreasing function of $L$ and vice-versa. The following is a summary of the observations made on Figs. 3 and 5.

$E_a$ and $C_l$ are unimodal functions of $L$, provided all other variables remain constant. Thus for any given parameter level combination there exists a value of $L$ at which efficiency is maximum; conversely $C_l$ is minimum. Two different cases can be discerned:

Case 1 ($L_{opt} \leq L_{max}$): the potential maximum is within the feasible range and occurs at an $L$ value which results in roughly equal $D_f$ and $R_f$.

Case 2 ($L_{max} \leq L_{opt}$): the potential maximum is outside the feasible range. The best practically attainable efficiency therefore occurs at $L = L_{max}$.

### 8.2. Basins

The BASin Design and EValuation software package (BASDEV, Jurriens and Boonstra, 1996) has been used in this analysis. The data used in the analysis is presented in Table 1. Generally, the range of the inlet flow rate, $Q_o$, into a unit width basin is bounded at the lower end by $Q_{min}$ (i.e. either the minimum required flow rate for advance to be completed or the flow rate required for adequate spread, whichever is greater) and at the other end by $Q_{max}$ (which is a function of soil erodibility and available supply). Figs. 6 and 8 present the response of the performance indices to variations in inlet flow rate, for $E_r = 100\%$ and $E_r = 90\%$, respectively. $E_a$ and $D_f$ are monotonic increasing and decreasing functions of inlet flow rate, respectively. The maximum application efficiency, $E_a$, and the minimum $D_f$ correspond to $Q_{max}$. Uniformity in a unit width basin increases as flow rate increases and is a decreasing function of length. Observe that for $E_r = 100\%$ and $Z_{min} = Z_r$, $E_a = \mathrm{DU}$ (Fig. 6). The above analysis holds true only when the basic assumption of ceteris paribus is satisfied.

Slightly different but similar observations can be made on the relationship between performance parameters and $L$ (Figs. 7 and 9). Note that all the curves in Figs. 7 and 9 are somewhat mirror images or lateral inversions of those presented in Figs. 6 and 8. What this means is that if an index increases with $Q_o$ then it is a decreasing function of $L$ and vice-versa.

## 9. Relationship between maximum $E_a$ and system variables ($Q_o$ and $L$)

### 9.1. Relationship between maximum application efficiency and flow rate

If $Q_o$ is constant throughout the time of application and $Z_r$ as well as $L$ are fixed and the target $E_r$ is known, Eq. (17) can be expressed as:

$$E_a = \frac{C_t}{Q_o t_{co}(Q_o)} \tag{20}$$

where $C_t$ = a constant equal to $E_r^* Z_r^* L$. In order to keep $C_t$ constant while $Q_o$ is varied, $t_{co}$ must also change. To reflect this, $t_{co}$ is expressed as a function of $Q_o$ in Eq. (20). Differentiating Eq. (20) with respect to $Q_o$ and setting the resulting expression to zero yields the following:

$$\frac{\mathrm{d}t_{co}}{\mathrm{d}Q_o} = -\frac{t_{co}}{Q_o} \tag{21}$$

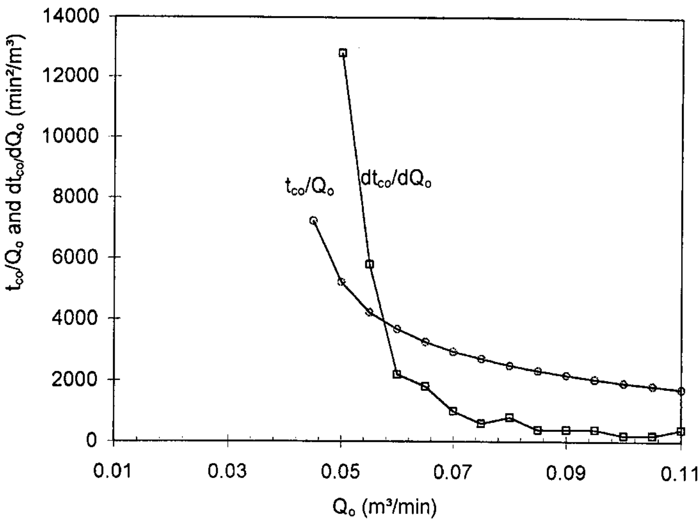


Fig. 10. Left-hand and right-hand sides of Eq. (21) against $Q_0$, for graded free draining channel ($E_r = 90\%$).

This is the condition for maximum $E_a$ with respect to $Q_0$. Expressed in a finite difference form this equation can be useful in locating the $Q_0$–$t_{co}$ combination that corresponds to the optimum $E_a$. The curves corresponding to the expressions on the left- and right-hand sides of Eq. (21) have been presented in Fig. 10 for furrow and in Fig. 11 for basin. The data used are the ones given in Table 1 ($E_r = 90\%$). Observe that in both cases: (1) the two curves are non-increasing convex functions of $Q_0$, (2) for graded free-draining channels the two curves intersect at the maximum $E_a$ only (showing that

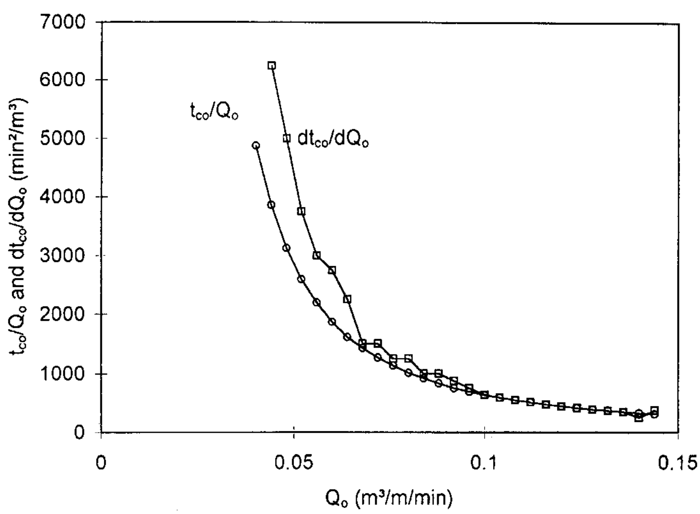


Fig. 11. Left-hand and right-hand sides of Eq. (23) against $L$, for basin ($E_r = 90\%$).

$E_a$–$Q_o$ curve is unimodal), and (3) for basins the two curves generally approach each other as $Q_o$ is increased, and become collinear at $E_a = 100\%$. Note that based on these observations, rules for optimizing $E_a$ with respect to $Q_o$ can be derived.

### 9.2. Relationship between maximum application efficiency and length

Using Eq. (17), for a given $Z_r$, $Q_o$, and target $E_r$ combination, $E_a$ can be expressed as:

$$E_a = C_t \frac{L}{t_{co}(L)} \tag{22}$$

where $C_t$ = a constant equal to $E_r^* Z_r / Q_o$. Differentiating Eq. (22) with respect to $L$ and setting the resulting expression to zero yields the following:

$$\frac{dt_{co}}{dL} = \frac{t_{co}}{L} \tag{23}$$

This is the condition for maximum $E_a$ with respect to $L$. The above equation can be expressed in a finite difference form, which is useful to locate the $L$–$t_{co}$ combination which corresponds to the optimum $E_a$. The curves corresponding to the expressions on the left- and right-hand sides of Eq. (23) have been presented in Fig. 12 for furrow and in Fig. 13 for basin. The data used are the ones given in Table 1 ($E_r = 90\%$). Observe that in both cases: (1) on average, the $dt/dL$ curve is a non-decreasing, convex function of $L$ (looking into the relationship between advance time and $L$, this observation does appear to be a general one); (2) the $t_{co}/L$ curve is a non-increasing convex function of $L$ for graded free-draining channels and a non-decreasing convex function of $L$ for basin; (3) for graded free-draining channels the two curves intersect at the maximum $E_a$

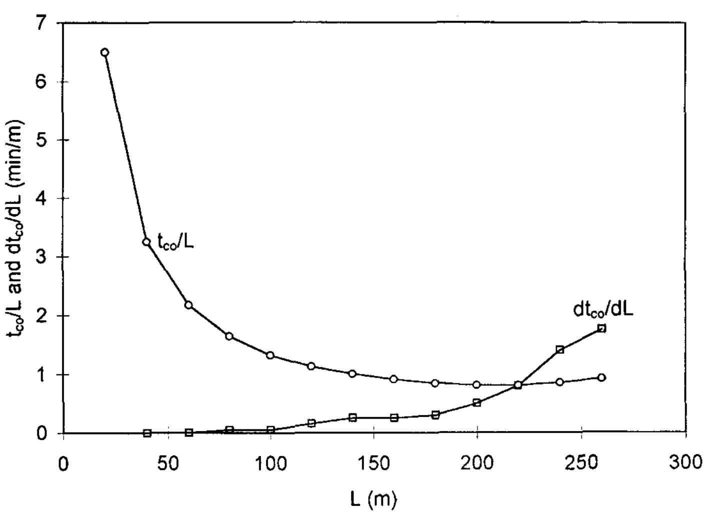


Fig. 12. Left-hand and right-hand sides of Eq. (21) against $Q_o$, for graded free draining channel ($E_r = 90\%$).

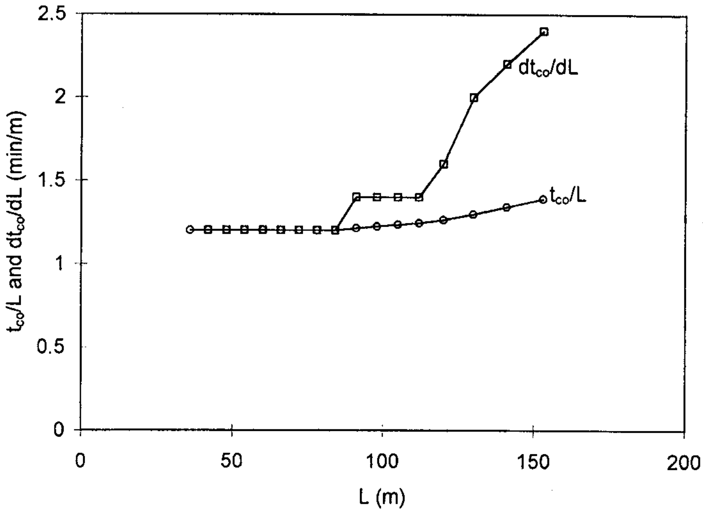


Fig. 13. Left-hand and right-hand sides of Eq. (23) against $L$, for basin ($E_r = 90\%$).

only (showing that the $E_a$–$L$ curve is unimodal); and (4) for basins the two curves diverge with increasing $L$, but they could, however, be collinear at low values of $L$ if $E_a = 100\%$. Note that from these observations rules for optimizing $E_a$ with respect to $L$ can be derived.

## 10. Summary and conclusions

Performance is defined in terms of a reference irrigation. In the context of on-farm irrigation water application, performance is event specific. In general the performance of a surface irrigation scenario (event) can fully be evaluated from three distinct but complementary perspectives: efficiency, adequacy, and uniformity. It is, however, sufficient to use only efficiency and adequacy terms to describe the performance of an irrigation event when $Z_r \leq Z_{min}$. Performance indices corresponding to each of those terms have been defined in terms of a perceived requirement and over a subject region. Terms that are related to irrigation water loss have been defined and their influence on system design and management decisions have been explored. The results of the study suggest that in graded free-draining channels, for a given field condition and target level of $E_r$, application efficiency is an unimodal function of $L$ or $Q_o$. In relation to level basins, the results indicate that $E_a$ is an increasing function of $Q_o$ and a decreasing function of $L$. Generally, uniformity is an increasing function of $Q_o$ and a decreasing function $L$. It appears from the cases studied that DU is a more sensitive parameter to changes in system variables than UCC. Based on the form of the equations used for evaluating both uniformity indices, one may observe that UCC is a good measure of uniformity over the entire length of the channel, whereas DU is a more appropriate indicator of the irrigation uniformity over the reach closer to the point where $Z = Z_{min}$.

Note that this is the section of the channel where the bulk of the non-uniformity in surface irrigation is concentrated.

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