Irrigation and Drainage Systems 9: 105–116, 1995

Improved 0–1 programming model for optimal flow scheduling in irrigation canals

ZHI WANG[1], J. MOHAN REDDY[2] & J. FEYEN[3]

[1] *Center for Irrigation Engineering, KU Leuven, Leuven, Belgium;* [2] *Department of Civil Engineering, University of Wyoming, Laramie, WY 82071, USA;* [3] *Center for Irrigation Engineering, KU Leuven, Leuven, Belgium*

Received 1 February 1994; accepted 16 September 1994

Key words: irrigation canals, optimization, scheduling, turnouts, zero-one programming

Abstract. An improved 0–1 programming model was presented for optimal flow regulation and optimal grouping and sequencing of outlets in irrigation distributaries, under restrictions of both the rotational period and the incoming flow rate into distributaries. The problem was solved using a commercially available 0–1 programming software package. The example computations indicated that this model could effectively provide a constant flow rate into the canal during most of the rotation period, and thus reduce the frequency of headgate operation. This formulation also minimized the accidental water wastage by appropriately sizing the canal cross-section.

1. Introduction

Irrigation distributary canals are designed usually with the assumption that the command areas among outlets or groups of outlets are nearly the same in size so that the water delivery amount and time will be nearly uniform. However, during the practical irrigation operation, owing to the ever changing cropping patterns, irrigation requirements, irrigation methods and technologies etc., the actual water demands and the time of delivery among outlets are dramatically varied, making it necessary to re-schedule the water deliveries before each rotation. The operational schedules may also need to be re-adjusted as the allowed incoming flow rate and/or inflow period changes.

Once the incoming flow rate and the intake time limit for a canal are prescribed and the running time of each outlet along a canal is estimated with the data of on-farm water requirement, water conveyance losses, and the discharge capacity of the outlet gate, the operation schedule of the outlets must be specified in order to operate either all the outlets simultaneously or in sequence. The former would necessitate a large canal capacity (economically undesirable) or a large flow rate that would probably exceed the capacity of the existing canal, while the latter would allow the use of a smaller canal (economically desirable) or a small flow rate in the existing canal, but the total running time of the canal might exceed the time limit of intake for the canal. An optimal operation schedule may exist, which would guarantee the supply of sufficient water to all the outlets within the time limit, using the most economical canal capacity. Once the canal operation schedule is decided, the inflow hydrograph into the canal and thus the headgate operation schedule can also be obtained.

The operation schedules can be prepared manually. But this traditional method has the disadvantage that the results may not be optimum and the procedure is time-consuming especially for large canals with large number of outlets. Thus it does not provide enough flexibility to make schedules efficiently and timely before each irrigation. With the aid of optimization techniques and computers, however, the plans can be prepared optimally, quickly and well before the start of each irrigation.

Suryavanshi and Reddy (1986) formulated, for the first-time, the outlet scheduling problem as a mathematical programming problem, and proposed a $0-1$ linear programming model for obtaining the optimal operational scheduling of irrigation canal outlets. However, a minor shortcoming in the formulation of the objective function was realized later on. Therefore, the objective of this paper is to provide an improved formulation of Suryavanshi and Reddy's model (1986), and to discuss the results obtained from application of the improved technique to two example problems.

2. Zero-one programming model

The following assumptions were made in the formulation of the outlet scheduling problem as a mathematical programming problem:

- all the outlets along the canal (be main, lateral, sub-lateral, tertiary, or farm canal) have the same discharge capacity by design. Though this is not a common situation, there are several irrigation schemes around the world where this situation is present. The formulation of the problem for unequal discharge rates from the outlets is more involved and is being investigated at present.
- during an irrigation rotation, once an outlet is opened, it runs continuously at its discharge capacity until the required volume of water is delivered; and
- the total discharge in the canal is composed of certain number of 'tube' flows (or sub-channels), and each 'tube' has the same net discharge capacity as that of each outlet along the canal.

Figure 1 shows the relationship between the 'tubes' and the outlets. Each outlet is free to draw water from any of the M 'tubes' during a given irrigation. And, each 'tube' can supply water to any number of N outlets in sequence. More

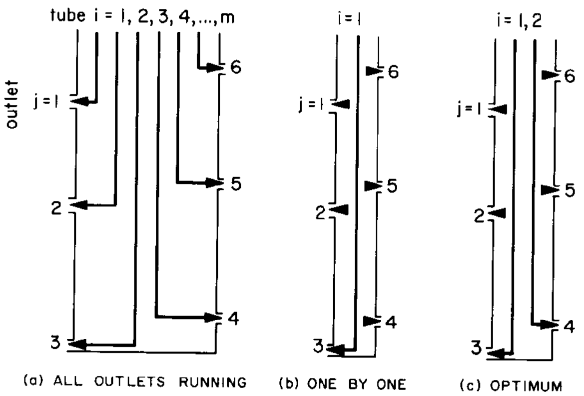

Fig. 1. Relationship between 'tubes' and outlets.

than one outlet may be open at any time during the rotation period. Those outlets connected to the same 'tube' belong to one group. When all the N outlets are open simultaneously, there would be N number of 'tubes' in the canal, e.g. $M = N$, as shown in Fig. 1a; and when all outlets are opened one by one, there would be only one 'tube' in the canal, combining all the N outlets into one group (Fig. 1b), and $M = 1$. When the given situation is in between the two extremes, the N outlets are divided into M groups ('tubes'), making $M < N$ as shown in Fig. 1c. Therefore, the capacity of the canal is indicated by the total number of the 'tubes', or the outlet groups, M, that are needed to supply the required quantity of water within the time-constraint specified.

2.1. Decision variables

A decision variable is defined as $X_{ij} = \{0,1\}$, in which i represents the number of the 'tubes' (or groups), and j represents the outlet number. When outlet j draws water from 'tube' i, then $X_{ij} = 1$, and $X_{ij} = 0$ when outlet j does not draw from 'tube' i.

2.2. Objective function

The objective is to find an operational schedule that minimizes either the construction cost of a new canal or the water conveyance loss of an old canal. Since the capacity or the flow magnitude of the canal is indicated by the number of 'tubes', this can be simply expressed in terms of the minimum number of 'tubes' in the canal:

$$Z = \min \sum_{i=1}^{M} C_i f_i(\sum_{j=1}^{N} X_{ij}); \qquad (M \leq N) \tag{1}$$

where C_i = the unit cost of construction or the unit conveyance loss of water for each 'tube'; Z = the minimum cost of construction or conveyance loss; and f_i = 'tube' (group) activation function which is defined as:

$$f_i(\sum_{j=1}^{N} X_{ij}) = \begin{cases} 1, & \sum_{j=1}^{N} X_{ij} \geq 1; \\ 0, & \sum_{j=1}^{N} X_{ij} = 0; \end{cases} \tag{2}$$

and X_{ij} = decision variable as defined before. Initially, M can be assumed equal to N. In Eq. 1, since all the 'tubes' are assumed to have the same capacity, then $C_1 = C_2 = \ldots = C_M$.

2.3. Constraints

According to the operational requirements of a canal, the following constraints can be written as:

Total operation time constraint: The total running time of the outlets in any group should not exceed the inflow period allowed for the canal. If t_j represents the running time of outlet j, the constraint can be expressed as:

$$\sum_{j=1}^{N} t_j X_{ij} \leq T, \qquad (i = 1,2,3,\ldots,M) \tag{3}$$

in which t_j = the running time of outlet j (days); and T = the total operation time available for the canal (days).

Outlet operation constraint: Each outlet, once opened, runs continuously for the required time:

$$\sum_{i=1}^{M} X_{ij} = 1, \qquad (j = 1,2,3,\ldots,N) \tag{4}$$

Incoming flow rate constraint: The sum of the 'tube' flows, e.g. total net flow rate of the canal, should not exceed the allowed gross inflow rate into the canal multiplied by the conveyance efficiency of the canal:

$$\sum_{i=1}^{M} q_i \cdot f_i \left(\sum_{j=1}^{N} X_{ij} \right) \leq Q \cdot \eta \tag{5}$$

in which q_i = net flow rate of 'tube' i, or the discharge capacity of the outlets (m^3/s); Q = incoming gross flow rate into the canal (m^3/s); and η = water conveyance efficiency of the canal.

0–1 constraint: Each decision variable takes a value of either zero or one:

$$X_{ij} = 0 \text{ or } 1 \tag{6}$$

Under some conditions, it may be desirable to group the outlets based upon the geopraphical location of the outlets, i.e., the first M1 number of outlets belong to group 1, the next M2 number of outlets belong to group 2, etc. If desired, these requirements can be added as additional constraints to the problem. However, this would limit the solution domain. In some cases, this might lead to a trivial solution. These constraints are not included here.

Eqs. (1) to (6) constitute a zero-one programming model for optimal scheduling of outlets on any canal. The solution to this model provides an operational schedule with optimum grouping of the outlets and the corresponding inflow hydrograph into the canal, thus providing dependable information for regulation of the headgate of the distributary canal as well as the scheduling of the lower level canals.

2.4. Solution

All the double-subscripted decision variables, X_{ij}, are first converted to equivalent single-subscripted variables, X_k, using the following relationship:

$$k = N(i-1) + j \tag{7}$$

where k = subscript number for variable X_k; i,j = number of the 'tube' and the outlet, respectively; and N = total number of outlets. Finally, X_k are reconverted to X_{ij} to reflect their operational meanings, using the following equation:

$$\begin{cases} i = INT(\frac{k}{N} + \frac{k}{k+1}) \\ j = k - N(i-1) \end{cases} \tag{3}$$

in which INT = integer function (for examples, INT(0.5) = 0, INT(1.2) = 1 and INT(1.99) = 1); i,j and N have the same meanings as defined before.

3. Application

Two examples were considered here. The first example was considered to show improvement of the present model over the Suryavanshi and Reddy's model (1986), while the second example was considered to show an application to a secondary irrigation canal (or lateral) with up to 26 outlets (or tertiaries). The total running time of each outlet was calculated using the data of on-farm crop water requirements, irrigated areas, field application efficiencies and water delivery efficiencies of the lower level canals.

Once formulated as a 0–1 programming problem, any of the several commercial software packages can be used to obtain the desired solution to the problem. Here, a commercial program called LINDO was used. An approximate solution can also be obtained using a spreadsheet program such as EXCEL or LOTUS123. It appears that a Neural Network formulation is also possible for this type of problem.

3.1. Comparing with Suryavanshi-Reddy model

Suryavanshi and Reddy (1986) used data from distributary 3 of the Meena branch canal in the Kukadi Irrigation Project in Maharashtra, India. The same set of data was used here to show the advantage of using mathematical programming solution over the traditional manual solution. The data are presented in Table 1.

The distributary had eight outlets (N = 8), and a maximum of eight 'tubes' or groups (M = 8). The unit cost of construction or the unit conveyance loss of water for each 'tube', C_i, was assumed to be the same. Therefore, $C_1 = C_2 = \ldots = C_8$. The zero-one programming model was formulated according to Eqs. (1) to (6). After substituting Eq. (2) into Eq. (1), we have:

Objective function:

$$Z = \min \sum_{i=1}^{8} C_i f_i\left(\sum_{i=1}^{8} X_{ij}\right) \tag{9}$$

Constraints:

Total operation time constraint (T = 6):

$$\sum_{j=1}^{8} t_j X_{ij} \leq 6, \qquad (i = 1,2,3,4,5,6,7,8) \tag{10}$$

Table 1. Canal and outlet data from Suryavanshi and Reddy (1986).

outlet j	1	2	3	4	5	6	7	8
t_j (day)	0.80	2.13	2.40	1.72	2.05	2.43	2.05	2.50

Total number of outlets: $N=8$;
Outlet discharge capacity: $q_i=q=30\ L/s$; and
total time available for the canal: $T=6$ *days.*

Outlet operation constraint:

$$\sum_{i=1}^{8} X_{ij} = 1, \qquad (j=1,2,3,4,5,6,7,8) \tag{11}$$

Incoming flow rate constraint: The maximum incoming flow rate Q was not given, so, this constraint was not considered here.

0–1 constraint:

$$X_{ij} = 0,\ 1 \tag{12}$$

The results obtained using the zero-one programming technique are as follows:

$$\begin{cases} X_{11}=1, \\ X_{12}=1, \\ X_{13}=1, \end{cases} \quad \begin{cases} X_{24}=1, \\ X_{25}=1 \\ X_{27}=1, \end{cases} \quad \begin{cases} X_{36}=1, \\ X_{38}=1, \end{cases} \qquad \text{and other } X_{ij} = 0$$

which means that the 8 outlets were sorted into 3 groups ('tubes'), with outlet 1, 2, 3 in the first group, 4, 5, 7 in the second group, and 6, 8 in the last group. The maximum net discharge through the headgate was determined as:

$$Q_{\max} = \sum_{i=1}^{3} q \cdot f_i\left(\sum_{j=1}^{8} X_{ij}\right) = 30\times 1 + 30\times 1 + 30\times 1 = 90\ell ps$$

The results from the Suryavanshi and Reddy's model (1986) are summarized as:

$$\begin{cases} X_{11}=1, \\ X_{14}=1, \\ X_{18}=1, \end{cases} \quad \begin{cases} X_{22}=1, \\ X_{25}=1, \end{cases} \quad \begin{cases} X_{33}=1, \\ X_{37}=1, \end{cases} \quad \{X_{46}=1, \qquad \text{and other } X_{ij} = 0$$

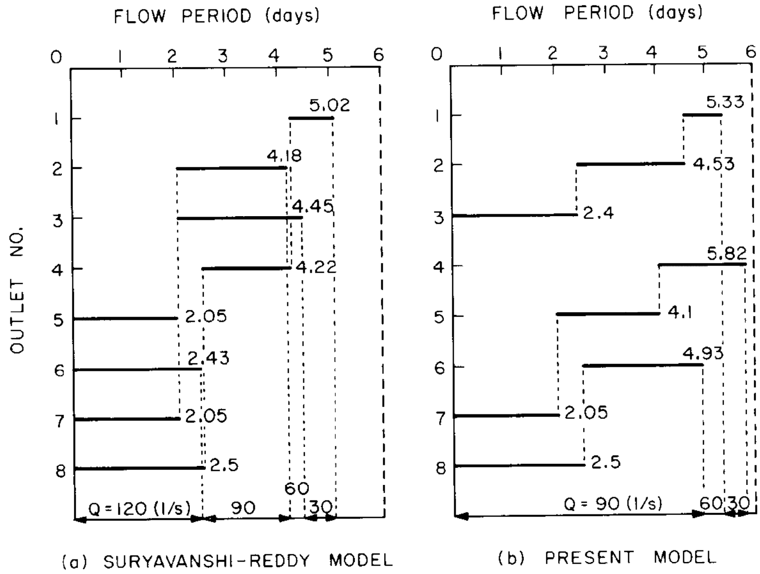

Fig. 2. Operational schedule for the Meena distributary.

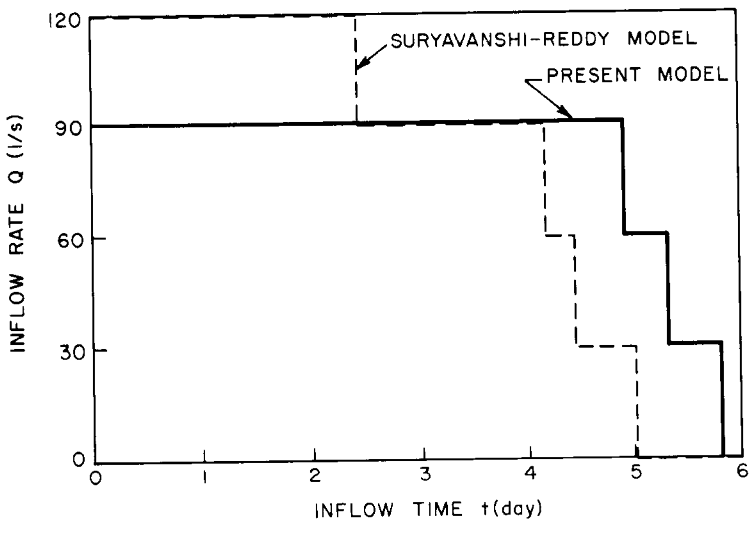

Fig. 3. Computed inflow hydrographs into Meena distributary.

thus four 'tubes' or groups were obtained with a net discharge rate, Q_{max}, of 120 ℓps through the headgate.

The two operational schedules for the Meena distributary 3 are shown in Fig. 2. The running sequence within a certain group was arranged according

Table 2. Inflow hydrograph into the distributary.

Flow rate into the distributary (fps)	Operating time (days)	
	Suryavanshi-Reddy model (1986)	Present model
120	0.00–2.43	
90	2.43–4.18	0.00–4.93
60	4.18–4.45	4.93–5.33
30	4.45–5.02	5.33–5.82
0	5.02–6.00	5.82–6.00

Table 3. Area and running time of tertiaries along Famen reach of secondary canal No. 11.

Tertiary No. j	area (ha)	time t_j (hour)	Tertiary No. j	area (ha)	time t_j (hour)
1	83	138	*14*	53	75
2	34	47	*15*	213	281
3	102	132	*16*	26	36
4	73	98	*17*	183	267
5	34	48	*18*	35	47
6	97	161	*19*	24	33
7	46	65	*20*	171	264
8	74	102	*21*	201	333
9	63	98	*22*	36	49
10	28	40	*23*	109	171
11	61	89	*24*	36	51
12	53	94	*25*	8	11
13	105	155	*26*	48	76

to a 'downstream to upstream' sequence. Other sequences may be applied according to specific conditions. The inflow hydrographs into the distributary, calculated on the basis of these two operation schedules, are given in Fig. 3 and Table 2.

It is clear from Figure 2 that the present model yielded less number of groups, and resulted in a better time balance within the allowed rotational schedule. It also provided a reduced discharge requirement and a more steady hydrograph into the canal resulting in less number of headgate settings and minimum canal cross-section.

3.2. Application to a secondary canal of Feng-Jia-Shan Irrigation District

Normally, there are several tertiaries on a secondary canal, and consequently a large number of groupings of the tertiaries (outlets) are possible. Therefore,

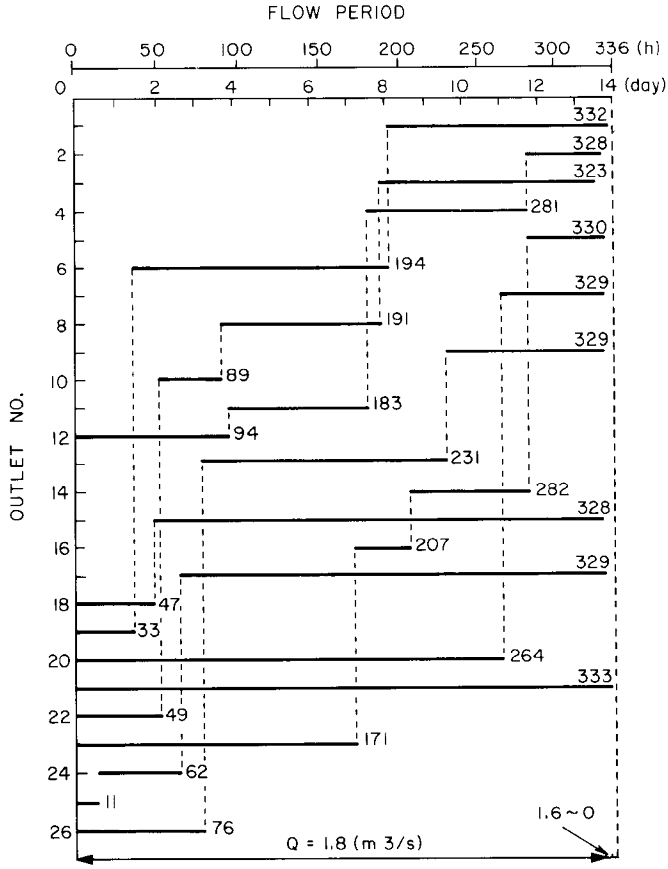

Fig. 4. Operational schedule for the Famen secondary canal reach.

it is almost impossible for the traditional manual methods to provide an optimal solution.

The secondary canal No. 11 of the North main canal, Feng-Jia-Shan Irrigation District, China, is a relatively large canal with a maximum discharge capacity of 2.8 m^3s and a total command area of 3930 ha. Administratively, the secondary canal No. 11 is managed by two sectors: the Famen sector which is responsible for the upper reach (2000 ha), and the Chenguan sector for the lower reach (1930). During the summer irrigation of 1987, the Famen sector was allowed to run for a maximum of 14 days with a maximum flow rate of 1.9 m^3/s. There are 26 outlets (tertiaries) in the Famen reach and each outlet has a discharge capacity of 0.2m^3/s. According to the gross field irrigation requirements and water conveyance efficiencies associated with each tertiary, the running time of each outlet was calculated as shown in Table 3.

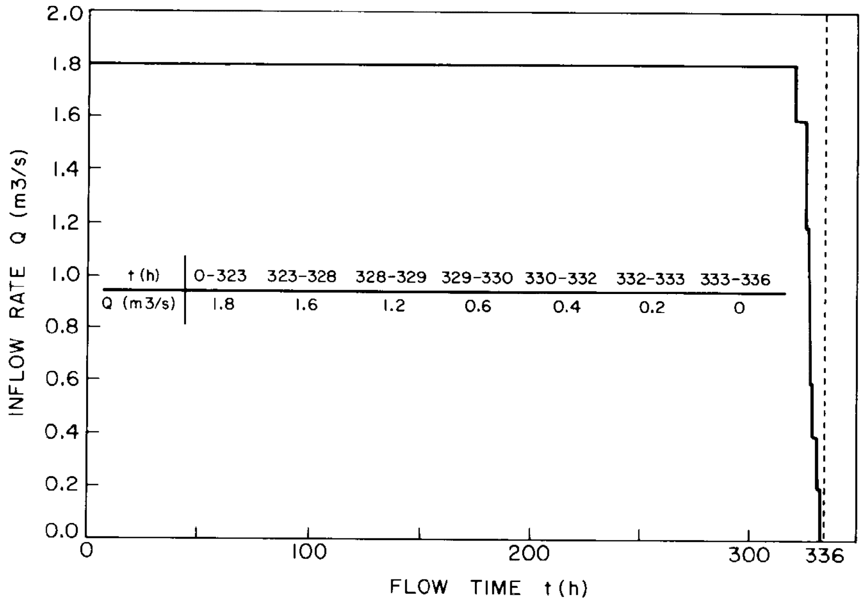

Fig. 5. Computed inflow hydrographs into Famen secondary canal reach.

Substituting the above information into the 0–1 programming model, Eqs. (1) through (6), the optimal operation schedule for the secondary canal No. 11 was obtained as shown in Fig. 4. The 26 tertiaries were optimally organized into a minimum of 9 groups, and the operation was accomplished 3 hours ahead of schedule. The inflow hydrograph into the secondary canal, as indicated in Fig. 5, shows that during the first 97% of the (323 h/333 h) rotation period, the flow rate into the canal was maintained constant, thus eliminating the need for headgate regulation during that period. Only during the last 10 hours, the gate was lowered 6 times to finally close it at the end of the rotation period.

Conclusions

An improved zero-one programming model for optimal flow rate scheduling of irrigation distributary canals was presented. Through use of micro-computers, the optimal grouping of the outlets was obtained first; then, the inflow hydrograph into the canal was calculated. However, the operational sequence of outlets within a certain group needs to be decided by the operators according to specific priorities.

The performance of the zero-one programming model presented here was compared with the original Suryavanshi-Reddy model (1986), and was applied to a secondary canal with up to 26 outlets. The improved model was found to be more effective especially when large number of outlets were to be optimally scheduled. Indirectly, the inflow hydrograph into the canal obtained using the optimization model was lower and less varied than the inflow hydrograph obtained using Suryavanshi and Reddy's model. Consequently, the canal flow rate and the number of headgate settings were reduced, providing a safer canal and gate operation scheme. The proposed method can also be used to determine the optimum discharge capacity of new canals.

References

Clemmens A.J. & Dedrick A.R. 1984. Irrigation water delivery performance. *Journal of Irrigation and Drainage Engineering*, ASCE,110(1): 1–13.

Fan M.Y. & Zhang J. 1986. *Fundamentals of the optimization techniques*. Qinhua University, Beijing, China.

Fletcher R 1981. *Practical methods of optimization 2: constrained optimization*. John Wiley & Sons, New York.

Nemhauser G.L. & Wolsey L.A. 1988. *Integer and combinatorial optimization*. John Wiley & Sons, New York.

Suryavanshi A.R. & Reddy J.M. 1986. Optimal operation schedule of irrigation distribution systems. *Agricultural Water Management*, 11: 23–30.

US Department of the Interior 1991. *Canal system automation manual*. Volume 1, Bureau of Reclamation, Engineering Research Center, Denver, Colorado.