

A Conceptual Model of Unstable Flow in Unsaturated Soil during Redistribution

William A. Jury,* Zhi Wang, and Atac Tuli

ABSTRACT

A conceptual model is presented that represents the development of unstable flow in uniform soils during redistribution. The flow instability results in the propagation of fingers that drain water from the wetted soil matrix until equilibrium is reached. The model uses soil retention and hydraulic functions, plus relationships describing finger size and spatial frequency. The model assumes that all soils are unstable during redistribution, but shows that only coarse-textured soils will form fingers capable of moving appreciable distances. Once it forms, a finger moves downward at a rate governed by the rate of loss of water from the soil matrix, which can be predicted from the hydraulic conductivity function. Fingers are assumed to stay narrow as a result of hysteresis, which prevents lateral diffusion. The draining front in the soil matrix between the fingers is assumed to cease downward movement because water pressure drops below the threshold water-entry matric potential. The threshold potential is not present in the conventional Richards equation of soil water flow, which explains why unstable flow is not predicted by widely used simulation model codes.

UNSTABLE FLOW DURING INFILTRATION in unsaturated porous media has been studied for many years and is known to be associated with a number of existing conditions, including vertical flow from a fine-textured layer into a coarse one (Hill and Parlange, 1972; Baker and Hillel, 1990), vertical flow into a compressed air phase (Peck, 1965), infiltration into water-repellent soil (Hendrickx et al., 1993), and two-phase flow involving two fluids of contrasting density and viscosity (Chuoque et al., 1959). More recently, it has been demonstrated to occur during infiltration into homogeneous soil at flux rates substantially less than the saturated hydraulic conductivity (Selker et al., 1992b; Geiger and Durnford, 2000), and during redistribution following infiltration in homogeneous soil (Diment and Watson, 1985; Wang et al., 2003a). Unstable flow is distinct from other forms of so-called preferential flow in that it is a fluid phenomenon whose extreme flow location is not a consequence of permeability variations in the porous medium.

Unstable flow in porous media was first analyzed by Saffman and Taylor (1958), who showed that perturbations in a wetting front between two immiscible liquids will develop into instabilities whenever the velocity of the invading fluid exceeds a threshold value. Raats (1973) and Philip (1975) demonstrated that the condition for instability at a wetting front in a porous medium was that the total hydraulic gradient behind the front is less than unity (i.e., the water pressure decreases toward the surface). Any effort to describe the fingering process must begin by characterizing finger geometry,

whose two most important attributes are finger diameter and the fraction of the total cross-sectional area occupied by fingers (hereafter called the *finger flow fraction*). Various attempts have been made to predict the finger diameter from soil properties and the water flow characteristics. They either involve an approximate equation that is a function of the sorptivity (Parlange and Hill, 1976) or include a macroscopic surface tension associated with the large-scale curvature induced by the radius of the finger (Chuoque et al., 1959; Glass et al., 1989a). Hydrostatic balance equations written at the wetting-front interface have periodic solutions that have a maximum unstable wavelength, and this is used to calculate the finger diameter. Both approaches involve associating one-half of the wavelength with the finger diameter. Parlange and Hill (1976) and Glass et al. (1989a) related the finger diameter to the soil's sorptivity S and the system flux ratio R_s by

$$d = \frac{2.4S^2}{K_s(\theta_s - \theta_i)} \left(\frac{1}{1 - R_s} \right) \quad [1]$$

where $R_s = i/K_s$ is the ratio of the infiltration or drainage rate to the saturated hydraulic conductivity. The expression of Chuoque et al. (1959) was adapted by Glass et al. (1989a) to produce a finger diameter of

$$d = a \sqrt{\frac{\sigma^*}{\rho_w g(1 - R_s)}} \quad [2]$$

where $a = 3.14$ (two-dimensional) or 4.8 (three-dimensional) is a geometric constant and σ^* is the effective macroscopic surface tension at the interface between the finger and the soil ahead of the advancing front.

Wang et al. (1998) replaced the effective surface tension with an expression involving the water-entry pressure head h_{we} of the wetting curve by using the capillarity equation $\Delta P = 2\sigma/R$ and defining an effective macroscopic curvature R^* at the interface. This modification produces the expression

$$d = a \sqrt{R^* |h_{we}|} \left(\frac{1}{1 - R_s} \right)^{1/2} \quad [3]$$

where R^* is the effective mean radius of curvature of the air-water interface at the wetting front. In fluid mechanics, R^* is assumed to be equivalent to the hydraulic radius of flow, which is defined as the ratio of the cross-sectional area of flow to the wetted perimeter (Bird et al., 1960), or the ratio of fluid volume in the pores to the contact area with the solid (Bear, 1972). For example, the hydraulic radius of a film flow, with film thickness h and width w , is equal to $wh/w = h$. The value of R^* tends to be relatively constant. Based on an analysis of the 24 existing laboratory and field studies of unstable flow reported in Wang et al. (1998), we concluded that $0.3 < R^* < 0.7$ cm, with a minimum estimation error at $R^* = 0.5$ cm. A thorough review of

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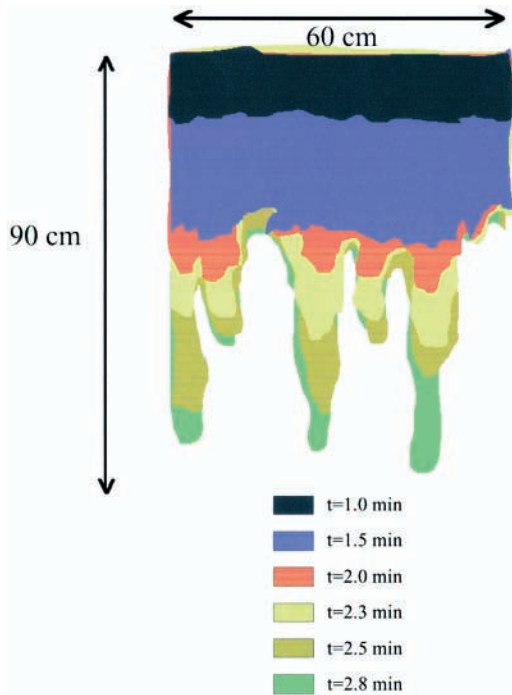


Fig. 1. Fingering occurring in homogeneous soil during redistribution in the Hele-Shaw experiments of Wang et al. (2003a). Region illustrated is a subset of the 1 by 1 m area of the chamber.

finger width experimental observations and models is given in de Rooij (2000).

The finger diameter is determined by two factors, the flow rate and the soil properties K_s and h_{we} . The fraction β of the total area at the front that the fingers occupy is more difficult to estimate because it depends on the rate of supply of water from the matrix to the finger, as well as on the flux through the finger. Glass et al. (1989b) analyzed Hele-Shaw cell data and came up with the empirical relation

$$\beta = 0.08 + 0.90\sqrt{R_s}; \quad r^2 = 0.955 \quad [4]$$

In contrast, Hillel and Baker (1988) calculated β at the interface between fine- and coarse-textured soil from a physical argument, assuming that all of the flux in the matrix is diverted to fingers after unstable flow begins, and that the flux through the finger is equal to $K(h_{we})$. With these assumptions, it is easy to show that

$$\beta = \frac{i}{K(h_{we})} = \frac{R_s}{K_r(h_{we})}; \quad K_r(h_{we}) = \frac{K(h_{we})}{K_s} \quad [5]$$

Note that Eq. [4] and [5] predict very different behavior of β as a function of R_s . In particular, the Hillel expression Eq. [5] is likely to underestimate the finger area fraction during redistribution when the matric potential opposes the flow.

Observations of the fingering process have repeatedly shown that fingers remain narrow as they propagate and persist for a long time after flow stops. Glass et al. (1989c) offered a physical explanation involving hysteresis in the matric-potential water content function (the water characteristic) for why the fingers don't disperse by lateral diffusion. The interior of the finger first wets to near saturation and then drains as the tip of the finger

moves downward. At the same time, a narrow zone of wetting is created at the outside fringe of the finger adjacent to the inner core. The interior of the finger thus follows the drainage loop of the water characteristic curve, whereas the fringe is on the wetting curve. Since at the same water content the drying curve is at a lower matric potential than the wetting curve, equilibrium is reached at different water contents inside the core and at the fringe, and the fingers remain narrow.

Instabilities form at a wetting or draining front when a portion of the front moves ahead of the rest and is fed by lateral flow from the surrounding soil matrix. An important factor in maintaining fingered flow, for reasons that will be discussed below, is the water-entry pressure head h_{we} , which is defined as the minimum matric potential head at the wetting front interface required to allow fluid to enter the dry portion of the medium below the wetting front. It has been measured experimentally in layered media by Baker and Hillel (1990) and in homogeneous media by Geiger and Durnford (2000) and Wang et al. (2000). Baker and Hillel (1990) also showed that it corresponds approximately to the matric potential at the inflection point of the wetting loop of the moisture release curve.

Although most of the effort to characterize fingered flow has been experimental, recent attempts have been made to model the transport process during fingered flow instability. Nieber (1996; Nieber et al., 2000) used a hysteretic form of the Richards equation together with a heavily biased numerical weighting scheme used to calculate hydraulic conductivity at the wetting front that essentially mimicked the water-entry matric potential by preventing water from moving into the dry region unless the wet zone was highly conducting. With this condition imposed, the Richards equation was able to predict the propagation of fingers. A different approach to modeling unstable flow was taken by Glass and Yarrington (1996), who used an invasion percolation algorithm that neglected viscous forces to create a complex pattern of gravity-induced fingers. The process is driven by pore filling in a medium composed of connected networks of pores.

The purpose of this paper is to use the known or postulated behavior of fingers during unstable flow to build a conceptual model of the fingering process during soil water redistribution. The emphasis of this approach is not simulation, but rather illustration of the main characteristics of fingering and its dependence on soil and management properties.

THEORY

Finger Shape and Propagation

Figure 1 shows a cross section of a portion of the draining front from a redistribution experiment conducted in a 1.0 m wide by 1.0 m deep by 0.1 m thick Hele-Shaw cell described in detail in Wang et al. (2003a). In this study, 5 cm of water was ponded on the surface of the cell containing a uniform dry coarse sand, and redistribution began at about $t = 1$ min. Several characteristics are evident in the figure. First, although perturbations in the shape of the front are present from the outset, instabilities do not form and cause fingers to grow until after redistribution begins. Second, once the fingering process

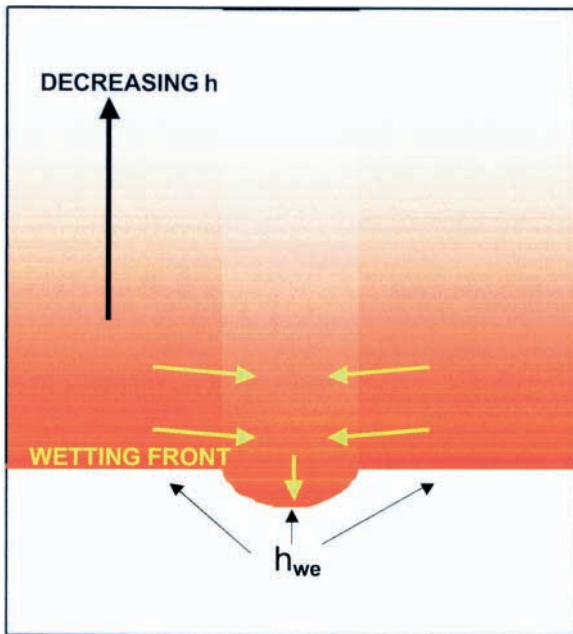


Fig. 2. Development of a fluid instability during redistribution, when the pressure distribution decreases toward the surface. When the front advances ahead at one location, the pressure distribution above it shifts downward, creating a lateral flow gradient from adjacent regions. Darker red color indicates wetter soil at higher matric potential.

starts, downward flow essentially stops in the matrix region between the fingers. Third, the initial finger area fraction β is substantial ($\sim 50\%$ in this experiment), but the fingers become thinner as the flow rate decreases.

Physical Explanation of Instability during Redistribution

Several conditions must be present for instabilities to form and propagate at a wetting or draining front in a homogeneous porous medium. First, a mechanism must be present to prevent water inside a propagating finger from easily entering the dry soil adjacent to it. As discussed in the introduction, Glass et al. (1989c) provided a plausible argument that hysteresis allows fingers to penetrate into the dry media without dispersing rapidly (Glass and Nicholl, 1996). Second, once a finger forms at the interface, it must be supplied by lateral flow of water from the surrounding soil matrix in the wetted zone as well as from the region directly above the finger. In addition, downward flow of water and advance of the draining front in the matrix zone between fingers must greatly slow down, so that water remains available for the finger. In order for these effects to occur, the dry soil region below the wetting front must have a threshold water-entry matric potential h_{we} , below which the wetting front will not advance. Fine-textured soils, with a wide range of pore sizes, do not have a distinct water-entry potential, but coarse-textured media made up primarily of large pores have been shown to possess a characteristic threshold for water entry (Geiger and Durnford, 2000; Wang et al., 2000). Because so much of the void space suddenly fills as this threshold is reached, the conductivity of the medium changes from negligible to high.

Figure 2 illustrates the formation and propagation of a finger during redistribution. During normal drainage, water pressure at the interface between the wet and dry zones is at the water-entry potential h_{we} , which allows water to enter the dry region below the entire draining front. As a perturbation forms, the depth of penetration becomes slightly greater at

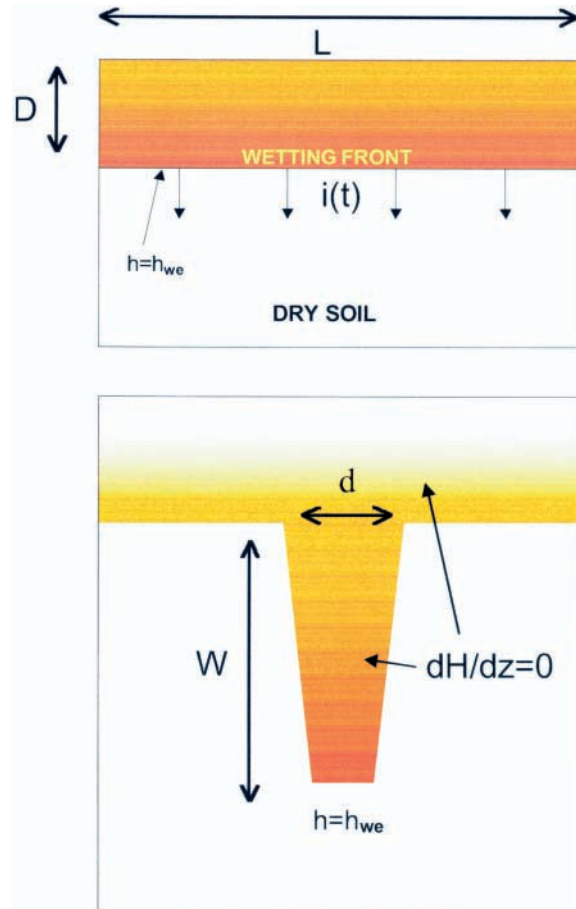


Fig. 3. Profile characteristics at the beginning (top) and end (bottom) of preferential flow. Darker red color indicates wetter soil at higher matric potential.

one point along the front, which shifts the water pressure distribution downward above that location. As a result, regions of the surrounding matrix begin to supply the zone above the finger, because the horizontal pressure distribution induces lateral flow. Subsequently, the water pressure in the surrounding matrix decreases, and the pressure at the wetting front drops below the water-entry pressure h_{we} , thereby stopping downward flow in the matrix region. The profile drainage then proceeds exclusively through propagating fingers and continues until flow stops. The finger may eventually dissipate, but this occurs on a time scale that is much longer than the propagation event (Glass et al., 1989c).

The Equilibrium Model

An approximate representation of the depth of penetration of a finger during redistribution may be achieved through an equilibrium analysis and the following assumptions:

- Infiltration proceeds at a constant flux rate i until the wetting front reaches a depth D below the surface, at which time redistribution begins.
- Fingers of diameter d form along the wetting front, and the wetted profile drains through the fingers.
- The fingers initially occupy a fraction β of the total cross section.
- Drainage continues until the finger and matrix are in hydrostatic equilibrium.

Figure 3 shows the draining profile at the beginning and end of the finger propagation event. Initially, the wetted soil

zone is filled with water from the infiltration process to a depth D , and has a water-content distribution $\theta(z)$ that depends on the soil and flux rate. A finger of diameter d forms and is supplied with water from the surrounding matrix, here represented as a zone of width L (or area L^2 in three dimensions), which defines the spatial frequency of the finger. The finger moves downward, with its leading edge remaining at the water-entry pressure h_{we} , until flow ceases everywhere in the matrix and finger domain. At this time, the profile has reached hydrostatic equilibrium ($dh/dz = -1$).

The final depth of penetration W of the finger below the original wetting front depth D may be obtained by mass balance. If we assume that the decrease in storage of the matrix is equal to the increase in storage of the finger, we obtain

$$A_m \int_0^D [\theta_{mi}(z) - \theta_{\infty}(z)] dz = \int_D^{D+W} A_f(z) [\theta_{\infty}(z) - \theta_{si}(z)] dz \quad [6]$$

where θ_{mi} is the initial water content in the matrix zone, $\theta_{\infty}(z)$ is the final water content in the finger and matrix zones, θ_{si} is the soil water content below the wetting front, and A_m and A_f are the cross-sectional areas of the matrix and finger domains, respectively (Fig. 2).

The initial water content or pressure distribution between the surface and depth D can be determined by traditional modeling with the Richards equation. It may be estimated approximately for the case of constant infiltration at a rate i by solving the steady flow equation

$$i = -K(h) \left(\frac{dh}{dz} - 1 \right); \quad h(D) = h_{we} \quad \theta_{mi} = \theta_w[h] \quad [7]$$

where $\theta_w(h)$ is the wetting branch of the water characteristic curve. The final water content distribution is the profile attained at hydrostatic equilibrium, and therefore is equal to

$$\theta_m(z) = \theta_d[h_{eq}(z)] = \theta_d[h_{we} - W - D + z] \quad [8]$$

where $z = 0$ at the surface and is positive downward, $h_{eq}(z)$ is the matric potential profile at equilibrium, and $\theta_d[h]$ is the drying curve of the water characteristic function.

The finger area fraction β is equal to $A_f(D)/A_m$, the portion of the matrix cross section at the draining front that is occupied by fingers. We will use the representation in Eq. [4] from Glass et al. (1989b) in our model calculations. During redistribution, the finger diameter shrinks because the flow rate is decreasing, eventually reaching zero. According to the finger diameter representation in Eq. [3], this means that the ratio of the final finger area to the initial finger area is $1 - R_s$. Assuming that the area declines linearly with depth (i.e., the finger is conical), we may model the area $A_f(z)$ in Eq. [6] by the expression

$$\frac{A_f(z)}{A_m} = \beta \left[1 - R_s \frac{(z - D)}{W} \right]; \quad D \leq z \leq D + W \quad [9]$$

With these assumptions, the final finger position may be calculated with Eq. [6] through [9] from the soil hydraulic properties, infiltration rate, and depth of wetting.

Dynamics of Finger Propagation

It is not clear a priori whether the rate of flow of water into the finger is limited by the rate of supply from the matrix or from the resistance within the finger domain. We tested the sensitivity of the rate of loss of water from the matrix to the finger geometry indirectly with the model HYDRUS-2D (Simunek et al., 1996), by using the profile from one-dimensional infiltration to depth L as the initial condition in a simulation where water was only allowed to drain through an aperture of width d . Simulations with different barrier widths

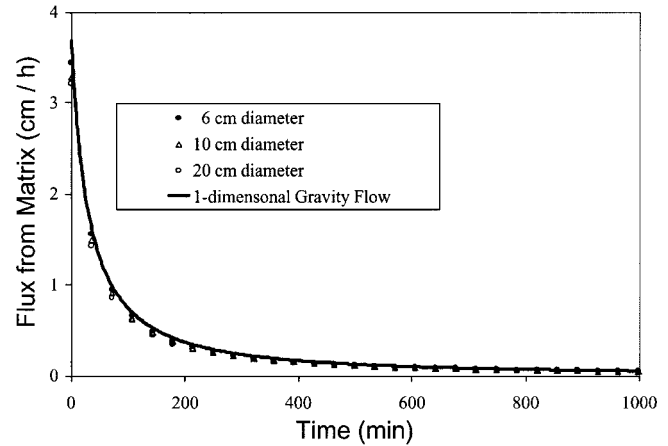


Fig. 4. Outflow simulated with the two-dimensional Richards equation through apertures of different diameter (symbols) and gravity flow model Eq. [10] (line). Initial condition is profile produced by constant flux infiltration to a depth of 10 cm.

produced nearly identical discharge rates from the aperture (Fig. 4), demonstrating that the rate of loss of water from the matrix governs the entry of water into the finger domain. Moreover, as shown by the line in the figure, this rate of loss is represented quite well by the gravity flow model

$$L \frac{d\theta_a}{dt} = -K(\theta_a) = q(t) \quad [10]$$

where θ_a is the average water content in the profile. The volume flow rate entering the finger from the matrix is given by $A_m q(t)$, and this is equal to the rate of increase of water stored in the finger. Thus, at time t

$$A_m \int_0^t q(t) dt = \int_0^{x(t)} A_f \theta_f(z,t) dz \quad [11]$$

where $x(t)$ is the depth of finger penetration below the wetting front at time t . The water content distribution $\theta_f(z,t)$ in the finger is determined by the supply rate $q(t)$ and the constraint that $\theta_f = \theta_{we}$ at the tip. As a rough approximation, we may assume that the water profile within the finger continuously adjusts to steady state, so that

$$q(t) = -K(h) \left(\frac{dh}{dz} - 1 \right); \quad h[x(t)] = h_{we} \quad [12]$$

In addition, the finger flow area at the tip of the finger will decline with time, and at any time t will be equal to (assuming $A_f \sim d^2$ and using Eq. [3])

$$\frac{A_f(t)}{A_m} = \frac{A_f(0)}{A_m} \frac{A_f(t)}{A_f(0)} = \beta \frac{(1 - R_s)}{[1 - q(t)/K_s]} \quad [13]$$

Equations [11] through [13] are solved iteratively at each time t for $x(t)$. The final position of x will correspond to the equilibrium position W calculated by Eq. [6] provided that drainage proceeds to equilibrium in a reasonable period of time.

Water-Entry Matric Potential h_{we}

Following Baker and Hillel (1990), we may estimate the water-entry matric potential h_{we} by determining the inflection point of the moisture release curve. Using the van Genuchten (1980) parameterization

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} = [1 + (\alpha|h|)^N]^{-M}; \quad M = 1 - \frac{1}{N} \quad [14]$$

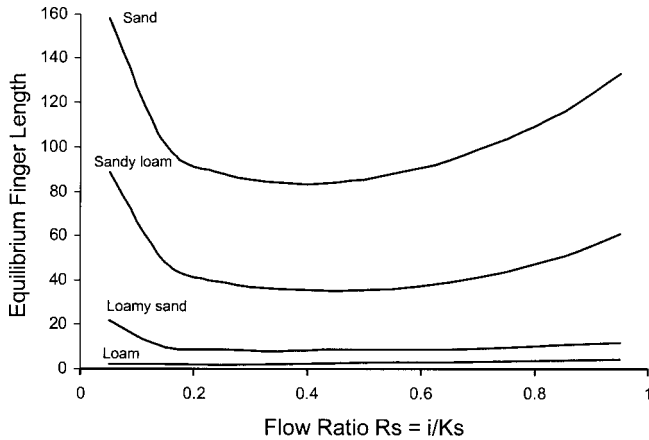


Fig. 5. Equilibrium finger depth reached during redistribution as a function of infiltration rate. Curves were calculated with Eq. [6] through [9].

we obtain the values

$$h_{we} = -\frac{M^{1-M}}{\alpha}; \quad \Theta_{we} = \left(\frac{1}{1+M}\right)^M \quad [15]$$

RESULTS

Figure 5 shows the predicted equilibrium depth as a function of R_s for fingers produced during redistribution following infiltration to a depth of 10 cm. Matrix potential–water content curves taken from HYDRUS-2D (Simunek et al., 1996) for four soils representing characteristic textures are used in the simulations (Table 1). Several features are prominent. First, although by assumption all soils are unstable during redistribution, the finger depth is negligible in the finer-textured soils. Second, maximum finger depths are large both at low infiltration rates and high ones, reaching a minimum at intermediate values of R_s . The reason for this behavior is made clear in Fig. 6, which shows a cross section of the final finger position for the sand at various flow rates. At large R_s , most of the cross-sectional area is taken up by the finger (see Eq. [4]), but there is a large amount of water stored in the 10 cm matrix profile because the infiltration rate nearly saturated the soil. In contrast, there is much less water stored during infiltration at the lowest R_s , but the finger occupies only a small part of the cross section and therefore receives a lot of water from the surrounding matrix and penetrates to great depth.

Figure 7 shows the dynamics of finger growth in the sandy soil for the case of $R_s = 0.25$, calculated using Eq. [11] through [13]. The flux $q(t)$ from the matrix into the finger was calculated with Eq. [10] using the $K(h)$ function

Table 1. Parameters of Eq. [14] through [15] for soils used in the simulations.

Texture	θ_s	θ_r	α	N	K_s	β	γ	h_{we}
Sand	0.43	0.045	0.145	2.68	29.7	0.315	3.24	-5.7
Sandy loam	0.41	0.057	0.124	2.28	14.6	0.300	3.07	-6.3
Loamy sand	0.41	0.065	0.075	1.89	4.42	0.212	3.03	-9.0
Loam	0.43	0.078	0.036	1.56	1.04	0.132	3.03	-14.4

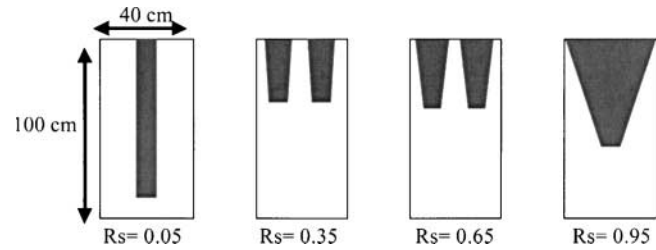


Fig. 6. Finger shapes formed in sandy soil during redistribution following infiltration at different flux ratios $R_s = i/K_s$.

$$K(h) = \frac{K_s}{[1 + (\beta|h|)]^\gamma} \quad [16]$$

Equation [16] was used in preference to the van Genuchten (1980) $K(\theta)$ function derived from the moisture release curve because the van Genuchten relation does not allow equilibrium to be reached in a physically reasonable time for coarse-textured soils, unless the medium is extremely wet. The values of β and λ were adjusted so that $K(h_{we}) = 0.125K_s$ and $K(-100) = 0.1 \text{ mm d}^{-1}$, which were deemed physically reasonable set-points for purposes of illustration. With these criteria, the dynamic finger growth model approaches the equilibrium value calculated by Eq. [5] within a few days.

DISCUSSION

Criteria for Instability and Finger Propagation

The notion that homogeneous soils may be unstable during redistribution is not widely accepted in mainstream soil physics, although it has been reported in the literature (Diment and Watson, 1985; Wang et al., 2003a). It has long been believed that capillary flow has a stabilizing influence on water movement, and that any perturbations in the advance of the wetting front that developed would be quickly damped out by lateral flow. In addition, prevailing theory would suggest that other parts of the draining front behind any finger in the process of formation would continue to advance, thereby depriving the finger of the water it would need to outpace the rest of the profile. Clearly that is not the case in Fig. 1, where the advancing fingers actually become narrower during their downward journey, and the matrix flow between the fingers virtually ceases. However,

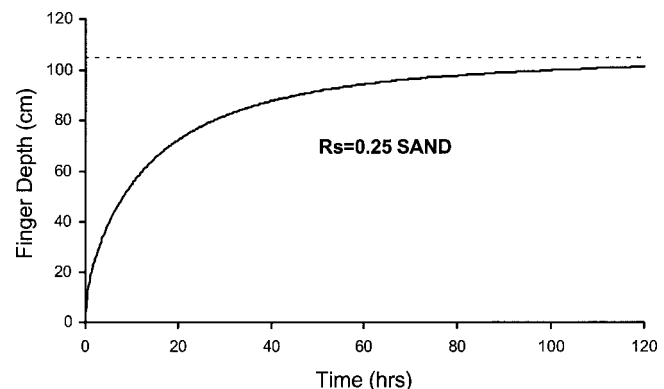


Fig. 7. Finger depth reached as a function of time for the sandy soil following an infiltration at $R_s = 0.25$, calculated with Eq. [10] through [13]. Equilibrium position is shown as a dashed line.

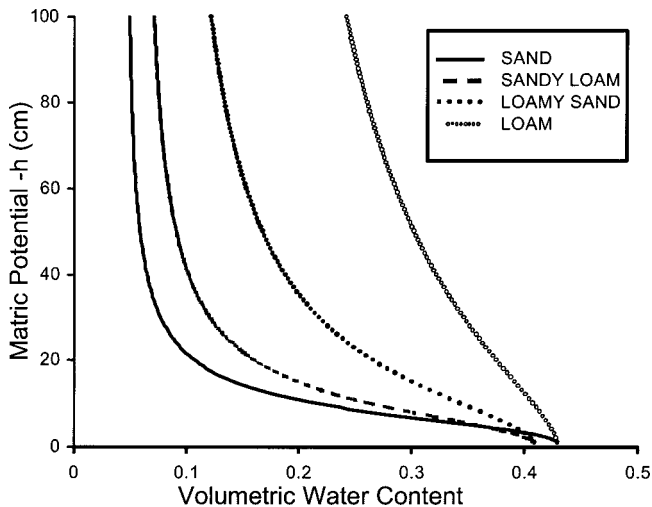


Fig. 8. Matric potential–water content curves calculated for the four soils, using the model Eq. [15] and the parameters in Table 1.

if we accept the hysteresis explanation for fingers remaining narrow (Glass et al., 1989c) and assume that the water pressure in the matrix drops below the water-entry pressure h_{we} , then fingers will form and propagate whenever the matric potential gradient opposes the flow. The water-entry pressure has been demonstrated to exist in soil in several experiments (Baker and Hillel, 1990; Geiger and Durnford, 2000; Wang et al., 2000) showing that water will not enter a dry medium until the liquid pressure at the point of entry is raised above a critical level, at which point infiltration suddenly begins.

However, the continuum description of water flow described by the Richards equation has no water-entry matric potential, because water will always flow from higher matric potential to lower, no matter what is the potential of the wetter region. To see that this is so, we need only examine Philip's solution to the equation for infiltration (Philip, 1969), where he shows that the sorptivity is a function of the difference in water content between the wet and dry regions, and that water will infiltrate into a completely dry medium at any wet region potential. Thus, in order for the Richards equation to model finger propagation correctly, it must include hysteresis to prevent finger widening and somehow insert a water-entry potential to prevent continued downward flow in the matrix. The only attempt to include the latter effect has been by Nieber (1996; Nieber et al., 2000), who calculated the conductivity at the wetting front by a weighting procedure dominated by the low conductivity in the dry zone below the front. Whether this procedure provides an adequate substitution for using the threshold matric potential directly is a question that must await further study.

Soils and Instability

Our simulations are based on assumptions that predict that all soils will be unstable during infiltration. However, as indicated by Fig. 5, finger propagation is only important in coarser-textured soils, because fingers in finer media are wider, contain more water, and occupy more of the total cross section than in coarse-

textured soil. As a result, the redistribution process will reach completion without much downward movement and the forward advance of fingers will be negligible. A key factor in a soil's susceptibility to deep penetration by fingers is the shape of the drying curve of the water content–matric potential function (Fig. 8). The equilibrium ($h + z = \text{const.}$) water content profile in the finger when movement stops is equal to the portion of the curve beginning at the tip of the finger where $\theta = \theta_{we}$ and extending upward. Since coarser-textured soils have little water remaining in them at relatively high matric potentials, most of the finger contains little water. The water content profiles for the finger shown in Fig. 5 have the same characteristic shapes measured in Selker et al. (1992a), with water content decreasing toward the surface.

Influence of Initial Water Content

The arguments to this point have focused on redistribution following infiltration into a dry medium. If the soil ahead of the wetting front contains water, the fingering process will be affected in several ways. The water-entry pressure will decrease, which will cause the fingers to become larger at a given flow rate. And, the capillary diffusion time scale will decrease because the fluid phase will have more continuity, with the consequence that fingering will dissipate more easily and the matrix flow between the fingers will be less likely to stop completely. As a result, the extent of fingering during redistribution will be less at a given infiltration rate as the water content ahead of the wetting front increases. These concepts are consistent with observations we made in a recent set of experiments in a Hele–Shaw cell (Wang et al., 2003a).

Finger Characteristics

The two most important features of fingering are the finger diameter d and the finger area fraction β or spatial frequency. Finger diameter has been predicted reasonably well with the equations given here, and with similar formulations discussed in the review by de Rooij (2000). In particular, Eq. [3] requires only the water-entry matric potential and the infiltration rate, and has estimated finger diameter satisfactorily both in the laboratory (Wang et al., 1998, 2003a) and in the field (Wang et al., 2003b). In this paper, we have assumed that finger diameter shrinks according to Eq. [3] during redistribution, producing a conical shape that is roughly similar to what we have observed in our experiments. Thus, we feel that this characteristic is predictable.

We are less sanguine about the predictability of the finger area fraction β . As we stated in the introduction, various authors have used quite different models for β , and the form we used (Eq. [4]) for our calculations is an empirical relation derived from laboratory data (Glass et al., 1989b). As shown in Fig. 1, the area fraction declines with time as the fingers shrink during redistribution, and it is difficult to resolve the exact fraction occupied by fingers at the outset. Additional experimentation under different conditions will be required to produce more accurate models of this key parameter.

CONCLUSION

We have used a mixture of physical arguments and empirical relations to produce a conceptual model of the fingering process during redistribution. Although the model is simplistic, it was able to demonstrate some useful conceptual features of fingering, and to show which soils have a strong susceptibility to fingering. The most critical features of soil that create and maintain fingering are hysteresis, which allow fingers to stay narrow as they advance, and the water-entry pressure, which quickly blocks the water in the soil between the fingers from moving downward once fingering begins.

The finger apparently does not form the limiting resistance to water discharge from the wetted soil matrix, so that it is possible to develop a flux rate into the fingers that is a function only of the soil characteristics and initial wetting status at the end of infiltration. The finger will grow until hydrostatic equilibrium is reached in the soil profile, although it may be moving at a negligible rate before that time.

Capillary effects can overcome the barrier formed by the water-entry pressure, but apparently operate on a time scale longer than the finger propagation time. As a consequence, fingers are able to move substantial distances before dissipating into the surrounding matrix.

The conventional Richards flow equation cannot predict the occurrence, transport, or final extent of fingers unless both hysteresis and some representation of the water-entry pressure are included. Without these effects, capillary diffusion will smooth out fingers at the moment of formation and also allow the water in the matrix between fingers to move downward.

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