Semivarigrams and Kriging

Dr. Zhi Wang
Department of Earth and Environmental Sciences
California State University, Fresno

Descriptive

- Similar to autocorrelations, but here we focus on the difference between pairs of observations separated by a lag h.
- Although measurements taken very close to each other tend to be similar, variations are generally highly irregular.

Relevant questions

- Should samples be taken randomly or at uniform, irregular intervals?
- How close together should samples be taken?
- Do variations differ with direction of sampling?
- Scale variant or invariant?
- How many samples are needed for a reliable semivarigram?

Semivariance

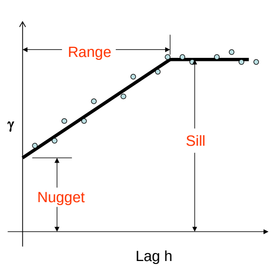
Semivariance:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[A_i(x_i) - A_i(x_i + h) \right]^2$$

N(h) is the total number of pairs of observations separated by h.

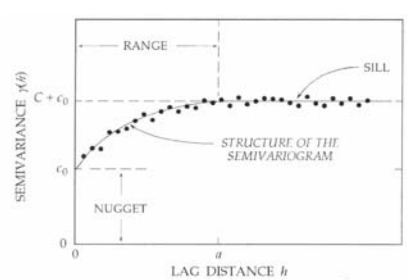
Semivarigrams

- Nugget variance: a non-zero value for γ when $h = 0$. Produced by various sources of unexplained error, or spatial variations occurring over smaller distances than the smallest sampling interval.
- Sill: for large values of *h* the variogram levels out, indicating that there no longer is any correlation between data points. The sill should be equal to the variance of the data set.
- Range: is the value of *h* where the sill occurs (or 95% of the sill). Mirror image of correlation length.

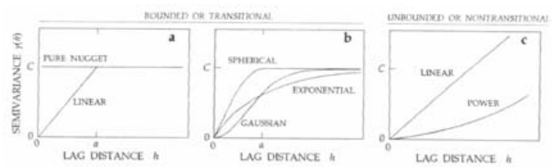


Sample size

- In general, 30 or more points are needed to generate a reasonable sample variogram.
- The most important part of a variogram is its shape near the origin as closest points are given more weight in the interpolation process.



Variogram models



Theoretical models of seven different kinds of semivariograms.

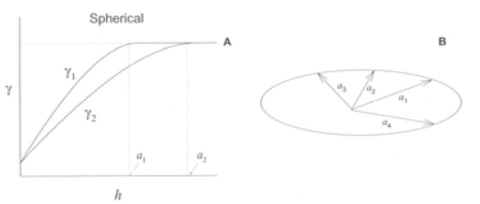
Variogram models

Equations for the seven semivariogram models

Model	Equation	Consideration
	Bounded or Transitional	
Pure nugget	$\gamma(h)=\begin{cases}0\\C\end{cases}$	$h=0$ $h>0$
Linear	$\gamma(h)=\begin{cases}Ch/a\\C\end{cases}$	$0\le h\le a$ $h>a$
Spherical	$\gamma(h)=\begin{cases}C\left[\frac{3h}{2a}-\frac{1}{2}\left(\frac{h}{a}\right)^3\right]\\C\end{cases}$	$0\le h\le a$ $h>a$
Exponential	$\gamma(h)=C[1-\exp(-h/a)]$	$h\ge 0$
Gaussian	$\gamma(h)=C\{1-\exp[-(h/a)^2]\}$	$h\ge 0$
	Unbounded or Nontransitional	
Linear	$\gamma(h)=mh$	$h\ge 0$
Power	$\gamma(h)=mh^{\beta}$	$h\ge 0; 1<\beta<2$

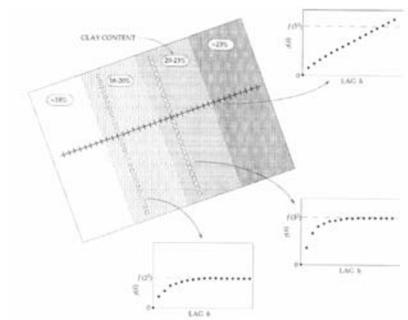
Directional dependence

- There may be higher spatial autocorrelation in one direction than in others, which is called anisotropy:



- Anisotropy is incorporated in the variogram model by means of a linear transformation.

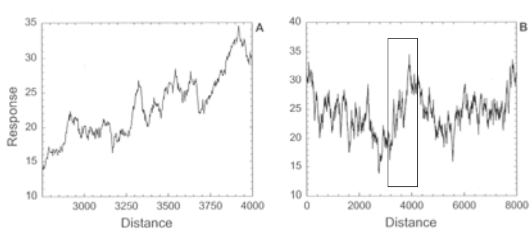
Directional dependence



Semivariograms derived from soil samples taken along three transects in a field having nonuniform distributions of clay content.

Drift (local trend)

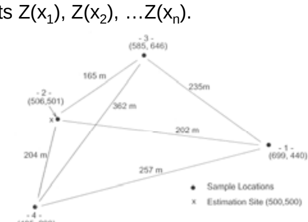
- Some attributes change in a systematic manner across the landscape:



In A there is an apparent trend in the values (changing mean). In B more data was collected and drift is no longer apparent (scale dependency, knowledge of the data).

Kriging

- Kriging is based on the idea that you can make inferences about a random function Z(x) given data points $Z(x_1)$, $Z(x_2)$, …$Z(x_n)$.



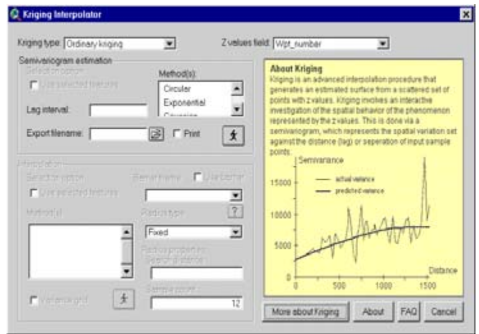
$Z(x) = m(x) + \gamma(h) + \varepsilon$"

3 components: structural (constant mean), random spatially correlated component and residual error.

Inference

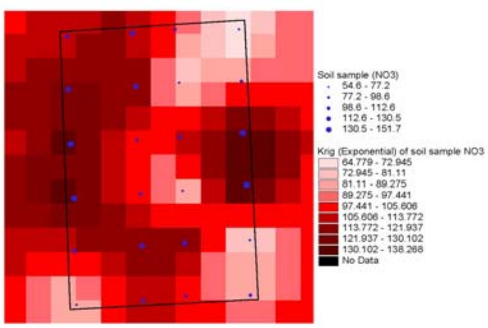
- Kriging produces the best linear unbiased estimate of an attribute at an unmeasured site, knowing only the variogram.
- Ordinary kriging: Most robust and most used
- Lognormal kriging: Ordinary kriging of the logarithms of the measured values (strongly positively skewed data that approximate a lognormal distribution)
- Universal kriging accounts for drift (local trend in variogrms).
- Punctual kriging: produces values for non-sampled points.
- Block kriging: gives values for inferred areas instead of inferred points. Estimates for blocks have lower variance because of the averaging of the small scale fluctuations of the function [Z(x)] over the area of the block.
- Co-kriging: uses 2 or more variables that are correlated between themselves in the estimation of values for one of them (e.g: soil bulk density and soil water content).

Examples from ArcView

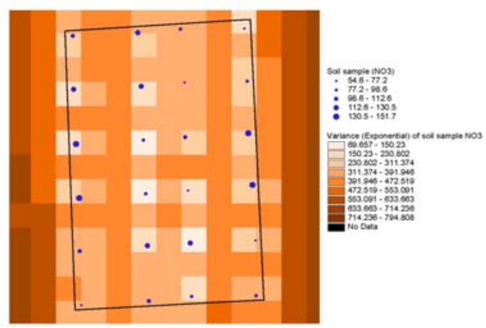


ESRI web site: Kriging interpolator 3.2 for ArcView Spatial Analyst

Example: kriging results



Example: variance map



Computer exercise

- Use GEOSTAT software to produce semivarigrams of soil attributes (data provided in the Excel data-file).
- Determine a best fit model of the semivarigram.
- Get data and redraw the semivarigrams in Excel, print results.