Hypothesis Testing

Two Sample t-tests:
Same or different?
Between-subjects t-tests

Topics

- The logic of hypothesis testing
- Z-test
- Z-score
- T-tests
  - One sample, two-tailed
  - One sample, one-tailed
  - Independent samples = between-subjects
  - Related samples = within-subjects

Between-subjects design

- A research design that uses a separate sample - a separate set of subjects - for each treatment condition (or for each population)
- = an independent-measures design

The basic approach

- We use the difference between sample means: $\left(\overline{x}_1 - \overline{x}_2\right)$
- As the basis for testing hypotheses about the difference between population means:

$$\left(\mu_1 - \mu_2\right)$$

Example

- Do the achievement scores of CSU students differ from the achievement scores of Nebraska students?

Example

- Do the achievement scores of CSU students differ from the achievement scores of Nebraska students?
- The null hypothesis
  - μ CSU = μ UN
- The experimental hypothesis
  - μ CSU >> μ UN

Review: the single sample t-test

$$t = \frac{\bar{x} - \mu}{\sqrt{\frac{s^2}{n}}}$$

Sample mean - Hypothesized pop. mean

Estimated Standard error of the Sample mean

The Differences between t-tests

$$t = \frac{\bar{x} - \mu}{\sqrt{\frac{s^2}{n}}} \qquad t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Single sample t-test

The t-test

Sample mean difference - Hypothesized pop. mean difference

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Between t-tests

Sample mean difference - Hypothesized pop. mean difference

Estimated standard error of the Sample mean difference

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Between t-tests

$$\frac{\text{Sample mean difference - Hypothesized pop. mean difference}}{\text{Estimated standard error of the sample mean difference}}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

The t-test

The 'pooled variance' is used twice

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

The pooled variance

$$s_p^{\,2} = \frac{SSX_1 + SSX_2}{df_1 + df_2}$$

Single sample variance
SS=Squared Sum

$$s^2 = \frac{SSX}{df}$$

Single t   Between-sjs t

- Everything is the same
- There's just twice as much of it

Single t:

$$\text{t} = \frac{\bar{x} - \mu}{\sqrt{\frac{s^2}{n}}}$$

$$s^2 = \frac{SSX}{df}$$

Between-sjs t:

$$\text{t} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^{\,2}}{n_1} + \frac{s_p^{\,2}}{n_2}}}$$

$$s_p^{\,2} = \frac{SSX_1 + SSX_2}{df_1 + df_2}$$

Recipe for hypothesis testing

- Step 1: State the hypothesis
- Step 2: Set the criterion for the decision
  - t critical & P(t critical)
- Step 3: Collect data
- Step 4: Compute sample statistics
  - t observed
- Step 5: Make a decision
  - If t observed is more extreme than t critical,
  - Reject the null hypothesis

Step 1, Sample problem

- State the alternatives in plain English
- H0, Null hypothesis:
- H1, Experimental hypothesis:

Step 1, Sample problem

- State the alternatives in plain English
- Ho, Null hypothesis:
  - CSU students and students at the Uncollege have the same average scores
- Ha, Experimental hypothesis:
  - CSU students have better scores than students at the Uncollege

Step 1, State the hypotheses

- Restate the alternatives mathematically
  - For a sample being compared to an INFERRED population mean:
- Ho, Null hypothesis
- Ha, Experimental hypothesis

Step 1, State the hypotheses

- Restate the alternatives mathematically
  - For a sample being compared to an INFERRED population mean:

Note:
You don't need to know the sample means.
The hypothesis concerns the difference between them

- H0, Null hypothesis
  - CSU scores = UN scores = ?
  - CSU scores - UN scores = 0
- H1, Experimental hypothesis

Step 1, State the hypotheses

- Restate the alternatives mathematically
  - For a sample being compared to an INFERRED population mean:
- Ho, Null hypothesis
  - CSU scores = UN scores = ?
  - CSU scores - UN scores = 0
- Ha, Experimental hypothesis
  - CSU scores - UN scores > 0

The experimental hypothesis is a directional hypothesis

Step 2

- Set the test criterion for the decision
  - Select an alpha level, P(T critical)
  - Find (look up) the value of T critical
- 2a: One-tailed or two-tailed?
- 2b: What alpha level will you use? (0.05)
- 2c: Use Table B.4 (also, pg 402)

2a, Sample problem

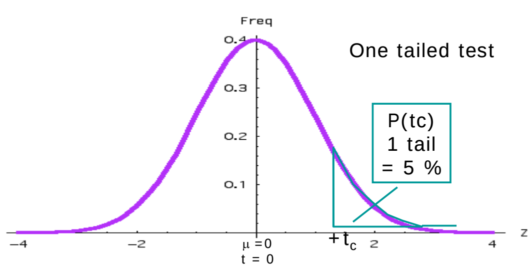
- He: $\mu \neq 0$
- Two-tailed test

- He: $\mu > 0$
- He: $\mu < 0$
- One-tailed test

Which is it?

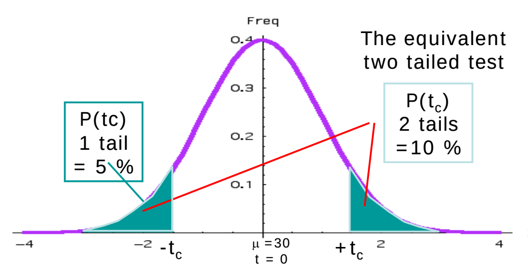
2b, Select an alpha level

- Alpha level, P(tc) = .05



Step 2b: Which is equivalent to

- A two-tailed alpha with twice the area



Step 2c: Use the table to find t_c

- df = degrees of freedom = 18
- .10 .05 .01 are values of alpha for 2- tailed tests
  - (Most tables are for 2-tailed alphas)
- When using a table for 2-tailed alphas,
- Double the alpha level
- For a 1-tailed test with alpha = 0.05,
- Use the column for a 2-tailed alpha = 0.10

Step 3, Get the data

CSU data	UN data
x bar = 25	x bar = 19
n = 10	n = 10
SSX = 200	SSX = 160

Total df = (n CSUF -1) + (n UN - 1)
= 10 - 1 + 10 - 1
= 18

Use the total df to find critical value in the table

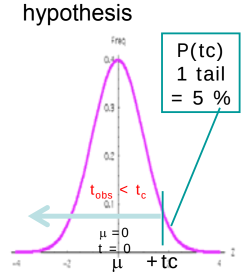
Step 4:
Calculate the between-sjs t

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

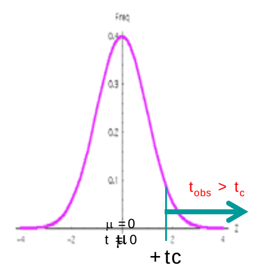
$$s_p^2 = \frac{SSX_1 + SSX_2}{df_1 + df_2}$$

Step 5, Make a decision

- If $t_{obs} < +t_c$, then FAIL TO REJECT the null hypothesis

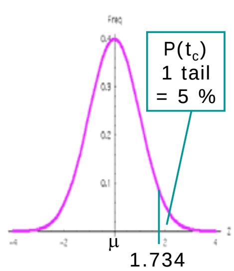


- If $t_{obs} > +t_c$, then REJECT the null hypothesis



Step 5, Result of the Analysis

- t observed = ?
- t critical = -1.734
- Decision?
- Do Wildcats stomp Cornhuskers?



How to report the results in APA format

- These data demonstrate that CSU students get significantly better scores than students at the Uncollege;
- t(18) = 3.00, p< 0.05, one-tailed.
- The likelihood of getting these data purely by chance is less than 5 in 100

Example 2

- CSU data
- x bar = 25
- n = 10
- SSX = 158

Ho: Δ (CSU-UN) = 0
Ha: CSU > UN

Use alpha = 0.05

- UN data
- x bar = 20
- n = 6
- SSX = 52

df = ?
tc = ?
Calc SSX1, SSX2, pooled variance, t observed

Results of example 2 in APA format

- These data demonstrate that CSU students get significantly better scores than students at the Uncollege;
- $t(14) = 2.50$, $p < 0.05$, one-tailed.
- The likelihood of getting these data purely by chance is less than 5 in 100

Exercises

Given the following two age dates (± 1 Standard Deviation) from buried marsh deposits on the California coast:

370 ± 60 Years BP
400 ± 60 Years BP

Do they represent the same time? How confident are you?