NUMERICAL ANALYSIS OF CIRQUES IN THE BEAR RIVER RANGE, NORTH-CENTRAL UTAH

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Twenty-two well-developed cirques are found in the Bear River Range, north-central Utah. Two cirque groups are recognized; those formed by valley glaciers and those formed by cirque glaciers. Comparison of these two groups affords an opportunity to develop inferences concerning late Pleistocene glacierization in this area.

Assumed relationships between glacier types and cirque attributes of morphology and location are examined by cluster analysis and principal component analysis. Detailed examination of attributes defines whether distinct sets of attribute values exist for each cirque group. This requires nonparametric tests applicable to data on a circular scale for the attribute of cirque orientation.

Results show that valley glacier cirques were subject to greater erosional efficiency. Valley glacier cirques are roughly twice the breadth of cirques formed by cirque glaciers. This greater erosional efficiency stems from variation in snow accumulation and retention between cirques in the two groups. Elevation and orientation are favorable for all cirques and provide no means to distinguish between groups. Location relative to wind direction is the distinguishing attribute. Cirques formed by valley glaciers occur on the leeward (east) side of the main crest or prominent peaks and ridges. Cirques formed by cirque glaciers are generally restricted to the windward (west) side of the main crest.

INDEX ENTRIES: Cirques, Glacierization, North-Central Utah, Bear River Range, Late Pleistocene, Morphology, Climatic Factors, Cluster Analysis, Numerical Analysis.

INTRODUCTION

Twenty-two well-developed cirques are found in the Bear River Range, north-central Utah (Figure 1). All cirques conform, in general terms, to descriptions commonly defining the cirque landform (Flint, 1971; Andrews, 1975). In terms of an operational definition all cirques correspond to the following description (Evans and Cox, 1974, p. 151):

A cirque is a hollow open downstream and bounded upstream by the crest of a steep slope which is arcuate in plan around a more gently sloping floor. It is glacial if the floor has been affected by glacial erosion while part of the headwall has developed subaerially and a drainage divide was located sufficiently close to the top of the headwall for little or none of the ice that fashioned the cirque to have flowed in from outside.

Cirque morphology varies, as shown in Table I. A bedrock lip or threshold was noted at some cirques. All cirques are located at the heads of pre-glacial, fluvial main canyons or their major tributary valleys that head along the main crest of the range (Figure 2). Horizontal to gently dipping carbonate bedrock is the lithologic and structural setting at all cirques.

The cirque population consists of two groups; those formed by cirque glaciers and those formed by valley glaciers. While it is recognized that mountain glaciers are initially cirque glaciers, environmental factors caused some to expand to valley glaciers while others remained in their initial form. In the Bear River Range, valley glacier cirques are identifiable by associated erosional modification of canyon bottoms and glacial deposits. These factors are absent at cirque glacier locations.

GEOLOGIC SETTING

Pleistocene glacial features have been identified and mapped in the southern part of the Bear River Range in north-central Utah. The range is part of the mid-latitude Rocky Mountains adjacent to the western boundary with the Basin and Range province. The Bear River Range is a north-trending range separating Cache Valley on the west from Bear Lake Valley on the east. The range consists of predominantly carbonate Paleozoic rocks broadly folded into a syncline on the west and an anticline

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on the east (J. S. Williams, 1958). The fold axis trends north-northeast. The main crest of the range generally coincides with the axis of the syncline. Summits along the crest range from 2717 m to 3042 m. The average elevation of the prominent named peaks along the crest is 2933 m. Glacial ice occupied the upper parts of seven major drainages in the Bear River Range. Quaternary landforms and deposits have been mapped and described by Young (1939), E. J. Williams (1964) and DeGraff (1976). E. J. Williams (1964) tentatively correlated moraines in lower Logan Canyon with the Bull Lake and Pinedale stages of the south-western Wind River Mountains of Wyoming (Holmes and Moss, 1955). Maximum glacier lengths (Bull Lake?) range from 0.8 km to 7.4 km with an average length being 2.7 km. The elevation of maximum down-valley ice limits ranged between 1658 m and 2609 m with an average elevation of 2142 m (DeGraff, 1976). Values for several tributary glaciers represent the distance to and elevation at the junction with the main Logan Canyon glacier.

COMPARISON OF CIRQUE GROUPS

A comparison of the two cirque groups provides insight into late Pleistocene glacial activity. This is useful for several reasons. Cirques can be viewed as a response to glacierization in a process-response model. Analysis of these two cirque groups yields inferences about late Pleistocene glacierization in this area. Additionally, it examines and characterizes cirques in a geographical area having few glacial geology studies.

It is initially hypothesized that the two cirque groups are distinguishable on the basis of morphology and location. Valley glacier cirques are expected to be bigger than cirques formed by cirque glaciers. It is assumed that valley glacier erosion was more effective and prolonged. Valley glacier cirques are expected to occupy locations favoring maximum snow retention through cooler temperatures and more effective shading. Therefore, valley glacier cirques should be larger, i.e., broader and deeper, at higher elevations, and have a restricted orientation generally between north and east compared to cirques formed by cirque glaciers.

If this hypothesized contrast is correct, classification of each cirque by morphology and location will correspond to the appropriate glacier type. Cluster analysis, a numerical taxonomy technique (Sneath and Sokal, 1973) was applied to evaluate this expected result. It was applied to the following variables: (1) minimum wall relief, (2) cirque breadth, (3) cirque floor elevation, and (4) potential annual insolation. The MINT cluster analysis computer program was used (Rohlf, 1973). Minimum wall relief is defined as the difference in elevation between the lowest point on the cirque rim and the cirque floor at the threshold. Cirque breadth is defined as the distance between the crest of cirque walls measured across the threshold area. Cirque floor elevation is the elevation of the cirque floor at the threshold. Potential insolation is the total annual insolation received at a given location. It is based solely on astronomic and geometric conditions affecting the amount of potential incoming radiation. Climatic factors such as number of cloudy days are not included. Potential insolation was substituted for orientation in
TABLE I
Cirque morphometry and characteristics.

<table>
<thead>
<tr>
<th>Name</th>
<th>Glacier type</th>
<th>Aspect (°)</th>
<th>Potential isolation (Th. Langley's)</th>
<th>Elevation (m ASL)</th>
<th>Minimum wall relief (m)</th>
<th>Breadth (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. South Fork of High Creek</td>
<td>V</td>
<td>338</td>
<td>206.6</td>
<td>2,682</td>
<td>73</td>
<td>732</td>
</tr>
<tr>
<td>2. Smithfield</td>
<td>C</td>
<td>160</td>
<td>279.8</td>
<td>2,682</td>
<td>49</td>
<td>732</td>
</tr>
<tr>
<td>3. South Fork of Smithfield</td>
<td>V</td>
<td>358</td>
<td>206.6</td>
<td>2,646</td>
<td>85</td>
<td>853</td>
</tr>
<tr>
<td>4. Birch</td>
<td>V</td>
<td>350</td>
<td>279.8</td>
<td>2,414</td>
<td>317</td>
<td>1,585</td>
</tr>
<tr>
<td>5. Green #1</td>
<td>C</td>
<td>120</td>
<td>270.6</td>
<td>2,841</td>
<td>18</td>
<td>366</td>
</tr>
<tr>
<td>6. Green #2</td>
<td>V</td>
<td>136</td>
<td>270.6</td>
<td>2,697</td>
<td>107</td>
<td>1,463</td>
</tr>
<tr>
<td>7. Upper Spring Hollow</td>
<td>V</td>
<td>60</td>
<td>219.1</td>
<td>2,633</td>
<td>122</td>
<td>1,951</td>
</tr>
<tr>
<td>8. Crescent Lake</td>
<td>V</td>
<td>104</td>
<td>246.4</td>
<td>2,670</td>
<td>49</td>
<td>1,463</td>
</tr>
<tr>
<td>9. Steep Hollow</td>
<td>V</td>
<td>100</td>
<td>246.4</td>
<td>2,633</td>
<td>85</td>
<td>1,951</td>
</tr>
<tr>
<td>10. Hell's Kitchen</td>
<td>V</td>
<td>100</td>
<td>246.4</td>
<td>2,487</td>
<td>91</td>
<td>1,341</td>
</tr>
<tr>
<td>11. Steam Mill</td>
<td>V</td>
<td>106</td>
<td>246.4</td>
<td>2,633</td>
<td>110</td>
<td>1,219</td>
</tr>
<tr>
<td>12. White Pine</td>
<td>V</td>
<td>80</td>
<td>246.4</td>
<td>2,560</td>
<td>171</td>
<td>2,560</td>
</tr>
<tr>
<td>13. Bunchgrass</td>
<td>V</td>
<td>126</td>
<td>270.6</td>
<td>2,560</td>
<td>24</td>
<td>1,341</td>
</tr>
<tr>
<td>14. Tony Grove</td>
<td>V</td>
<td>100</td>
<td>246.4</td>
<td>2,451</td>
<td>134</td>
<td>2,438</td>
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<tr>
<td>15. South Fork of Cottonwood #1</td>
<td>C</td>
<td>134</td>
<td>270.6</td>
<td>2,719</td>
<td>61</td>
<td>732</td>
</tr>
<tr>
<td>16. South Fork of Cottonwood #2</td>
<td>C</td>
<td>132</td>
<td>270.6</td>
<td>2,646</td>
<td>98</td>
<td>732</td>
</tr>
<tr>
<td>17. Cottonwood</td>
<td>C</td>
<td>90</td>
<td>246.4</td>
<td>2,573</td>
<td>49</td>
<td>853</td>
</tr>
<tr>
<td>18. Spring Hollow</td>
<td>V</td>
<td>360</td>
<td>206.6</td>
<td>2,268</td>
<td>245</td>
<td>975</td>
</tr>
<tr>
<td>19. Mill Hollow</td>
<td>V</td>
<td>2</td>
<td>206.6</td>
<td>2,597</td>
<td>146</td>
<td>975</td>
</tr>
<tr>
<td>20. Dry (South)</td>
<td>C</td>
<td>348</td>
<td>206.6</td>
<td>2,585</td>
<td>37</td>
<td>610</td>
</tr>
<tr>
<td>21. Providence</td>
<td>V</td>
<td>126</td>
<td>270.6</td>
<td>2,658</td>
<td>24</td>
<td>1,219</td>
</tr>
<tr>
<td>22. Leatham Hollow</td>
<td>V</td>
<td>90</td>
<td>246.4</td>
<td>2,134</td>
<td>61</td>
<td>1,463</td>
</tr>
</tbody>
</table>

*C—Cirque glacier; V—Valley glacier.

The cirque is at the head of the main canyon, unless the tributary name is noted.

In order to fit the scalar constraints of the cluster analysis technique, Potential insolation values were obtained by using the same slope and latitude for different aspects in published tables (Frank and Lee, 1966). All variables were on an interval scale and not standardized. The resemblance measure employed was euclidean distance (Sneath and Sokel, 1973, p. 124). Clustering was accomplished by employing the unweighted pair-group method using arithmetic averages (UPGMA). The resulting dendrogram shows six groups at a similarity level of 1.3 (Figure 3). Defining the dendrogram groups at a particular index of similarity is a standard judgmental decision called for in cluster analysis. The decision attempts to define as few meaningful groups as possible at the lowest index value. The first group includes all cirques formed by cirque glaciers and two cirques formed by valley glaciers. The remaining five groups are composed of valley glacier cirques.

The expected contrast between the two cirque groups does not completely agree with the observed clusters. Examination of the attributes is necessary to discern the basis for this disagreement. An initial effort involves determining which attributes were the principal influence on the resulting clusters. The clustering data were used with principal component analysis (Davis, 1973, p. 478). The data were standardized. Therefore, the resemblance matrix consists of the correlations (r) among the attributes rather than variance-covariance. Examination of the resulting eigenvectors shows that 98 percent of the total variance among sample points was explained by two of the four principal components. Because the resultant component scores are uncorrelated to the original input, it is necessary to compare plots of the original standardized data and component scores to determine which attributes contain the variance.

Comparison of plots identified minimum wall relief and cirque breadth as the attributes accounting for most of the variance. Minimum wall relief
accounts for 70 percent and cirque breadth accounts for 28 percent of the variance among sample points. Clearly, cirque morphology controlled the resulting clusters. These two attributes need to be analyzed to understand this result.

CIRQUE MORPHOLOGY

The hypothesis of contrast between cirque groups assumes that each cirque group has a distinct set of values for each attribute. This assumed uniqueness must be tested to establish the actual influence of the cirque morphology attributes. Initially, the equality of variance is tested by an F-test. Where the variances are not significantly different, the equality of means is tested by the Student T-test (Davis, 1973).

Tables II and III show the results of testing equality of variance and means for minimum wall relief and cirque breadth. The null hypothesis of equality of variance is rejected for both attributes. The null hypothesis of equality of means can be rejected only for cirque breadth. It is concluded that cirques formed by both valley and cirque glaciers have a common population of minimum wall relief values. The two cirque groups can be distinguished by distinct sample populations of cirque breadth. The mean value for valley glacier cirque breadth is 1470 m. This is approximately twice the mean value of 671 m for the cirque glacier cirque breadth. Clustering into cirque glacier and valley glacier cirque groups appears controlled by the distinct set of cirque breadth values. Minimum wall relief values add an inconsistent variation to the cirque clustering. This
FIGURE 3 Cluster analysis dendrogram of cirque and valley glacier cirques.

TABLE II
F-test of morphologic attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Degrees of freedom</th>
<th>Variance</th>
<th>F-value</th>
<th>Ho</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wall relief</td>
<td>15</td>
<td>5</td>
<td>8,642.1</td>
<td>9.41</td>
</tr>
<tr>
<td>Breadth</td>
<td>15</td>
<td>5</td>
<td>260,992.5</td>
<td>11.10</td>
</tr>
</tbody>
</table>

F table = 6.43, α = 0.025 (2.5%), V (Valley) and C (Cirque) are glacier type associated with cirque.
examination of cirque morphology attributes provides some inferences about ice erosion. Over-deepening of cirques appears equal between the two groups. Studies indicate that over-deepening by cirque glaciers results from abrasion and plucking (McCall, 1960; Weertman, 1971). It is suggested that over-deepening by the initial cirque glacier stage of valley glaciers was generally unmodified by the later valley glacier occupancy. Widening of a cirque results from subaerial erosion of sidewalls and backwalls. This erosion is enhanced by sapping, abrasion, and plucking along the lateral edge of the glacier (Andrews, 1975). Valley glacier cirques may represent locations where this erosion operated for a longer period of time and/or at greater intensities than cirque locations occupied only by cirque glaciers.

GLACIAL DEVELOPMENT

The two existing cirque groups result from corresponding differences in erosional efficiency between cirque and valley glaciers. This differing erosional efficiency and glacier type are assumed to result from variations in environmental factors influencing snow accumulation and retention at cirque locations. Environmental factors influencing the amount and rate of snow accumulation and retention include the elevation relative to the snowline, orientation relative to shading of the accumulation area, and location relative to wind direction. Other environmental factors that might be considered are preglacial relief, structure, and lithology. These factors are not considered significant due to their nearly identical circumstance at all cirque locations.

Cirques form at or near the orographic snowline (Flint, 1971). A rough estimate of the late Pleistocene orographic (or composite) snowline can be made by using elevations of cirque floors (Flint, 1971; Porter, 1964; Peterson and Robinson, 1969). Whether an elevation difference exists between cirque groups must be determined in the same manner as cirque morphology attributes. Elevation sample populations yield a calculated F-value of 2.93. Using the same significance level and table value as Table II, it is clear that the null hypothesis cannot be rejected. There is no evidence to conclude that the elevation values for each cirque group were drawn from different populations. Cirques formed by both glacier types developed within the same range of elevations. The mean elevation for all cirques in the Bear River Range is 2579 m, ranging from 2134 m to 2841 m. The mean value represents an approximate estimate of the late Pleistocene orographic snowline in this area. It falls within the late Pleistocene orographic snowline elevation range of 2591 m to 2743 m estimated for this region (Flint, 1971, p. 473). Elevation relative to the snowline does not account for the differing erosional efficiency and glacier type found in the Bear River Range.

Cirque orientation determines snow retention by controlling solar radiation (Embleton and King, 1975). Orientations favoring optimum shading distinguish some present-day, glacially occupied and ice-free cirques (Andrews and Dugdale, 1971; L. D. Williams, 1975). Comparing the sample population values for orientation data involves using data on a circular scale. Figure 4 shows all cirque orientation values. Because the distribution

![Figure 4](https://example.com/figure4.png)

**FIGURE 4** Rose diagram of all cirque orientations.
is not clearly unimodal and symmetric, a nonparametric test is applied to determine mean direction, confidence interval, and equality of mean directions. Watson's $U^2$ test was used to evaluate equality of direction means between cirque groups (Batschelet, 1965, p. 35). This is a nonparametric test requiring only an assumption of a continuous circular distribution. Following this procedure, a calculated value of $U^2 = 0.108$ where $n = 16$ and $m = 6$ is obtained. The table value is 0.182 (Batschelet, 1972, p. 84). Therefore, it cannot be concluded that the two populations are significantly different. The mean direction is the same for both cirque groups. The mean direction is tested by using the so-called sign test (Batschelet, 1965, p. 31). The median angle is determined to be $100^\circ$ with a confidence interval of $0^\circ$ to $132^\circ$ at a ten percent significance level. This median direction is in the optimum range for effective shading within the cirque accumulation area. Clearly, all cirques in the Bear River Range are oriented favorably for glacierization. Differing erosional efficiency and glacier type is not attributable to cirque orientation.

Some studies have shown cirque distributions in which elevation and orientation have a combined effect that groups cirque within an area (Seddon, 1957; Andrews, Barry, and Drapier, 1970). Figure 5 shows a polar diagram of all Bear River Range cirques. Inspection shows no pattern for these combined attributes which would account for differing snow accumulation and retention between cirque groups.

Wind direction resulting in snow drifting to shel-
tered leeward slopes enhances accumulation and influences cirque development (Embleton and King, 1973; Andrews, 1965; DeGraff and Zsiray, 1978). All Bear River Range cirques are found along the main crest of the range. Except for the valley glacier cirques at Birch, Smithfield, High Creek, and Green Canyons, all valley glacier cirques are on the leeward or east side of the main crest. The cirques formed by cirque glaciers are all on the windward or west side of the main crest, except the three cirques in Cottonwood Canyon. Using the Fisher’s exact test, 2 × 2 case, it is possible to test the independence of location (windward, leeward) from the attribute of glacial cirque type (valley glacier, cirque glacier). This test computes a cumulative probability to be compared to a corresponding table value (Lewis, 1977, p. 81). Table IV summarizes the objects and attributes.

**TABLE IV**

<table>
<thead>
<tr>
<th>Location</th>
<th>Attribute</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Valley</td>
<td>Cirque</td>
</tr>
<tr>
<td>Leeward</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>Windward</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>6</td>
</tr>
</tbody>
</table>

This is set up as a one-tailed test. The cumulative probability is 0.231 at a significance level of five percent compared to the table value of 2.032. Therefore, we reject the null hypothesis that cirques on the leeward side have the same or less likelihood of being valley glacier cirques than cirque glacier cirques. Valley glacier cirques are preferentially found on the leeward (east) side of the main crest. In summary, wind-drifted snow is responsible for the differing accumulation and retention resulting in the two cirque groups. This circumstance is enhanced by the favorable orientation for maximum shading effect of leeward slopes.

A study of optimum glacier location related to cirque form provides an explanation for the few cirques which are exceptions to this general conclusion (Graf, 1976). Graf (1976) noted that present-day, glacially occupied cirques among empty cirques had certain topographic attributes. A pass or opening to the windward side which increases snow accumulation by channelling wind into a cirque. Additionally, a peak or prominent ridge on the southwest or south rim of the cirque increases shading from solar radiation. Some drifting to the leeside of the peak or ridge is probably involved. Each of the four valley glacier cirques (Figure 2, No. 1, 3, 4, and 5), on the windward side of the main crest, have these topographic features. Clearly, cirques formed by valley glaciers occur on either the leeward (east) side of the main crest or of prominent peaks and ridges. On the leeward side of the main crest, the three small cirque glacier cirques in Cottonwood Canyon (Figure 2, No. 15, 16, and 17) have a related explanation for their anomalous location. These cirque glaciers were within 4 km of, and downwind from, the windward side valley glaciers at Green, Birch, and South Fork of Smithfield Canyon. Inasmuch as these valley glaciers were to the west and southwest of the Cottonwood cirque glaciers, it would appear that sufficient snow was retained on the windward side of the main crest to suppress expected full glacial development at the Cottonwood cirques.

**CONCLUSIONS**

Cirques in the Bear River Range can be divided into two groups; those formed by valley glaciers and those formed by cirque glaciers. This distinction is based on the association of canyon bottoms modified by ice erosion and glacial deposits only with valley glacial cirques. Also, lateral erosion was more efficient in valley glacier cirques. Valley glacier cirques are roughly twice the breadth of cirques formed by cirque glaciers. Overdeepening by the initial cirque glacier stage of valley glaciers appears to be generally unmodified by later valley glacier occupancy.

The two groups reflect corresponding differences in erosional efficiency stemming from variation in snow accumulation and retention. Orientation and elevation for all cirques are within a range favorable to late Pleistocene glacierization. No statistically significant difference exists between cirque groups for the sets of values of either attribute. However, location based on relation to wind direction does distinguish the two groups. Cirques formed by valley glaciers occur on the leeward (east) side of the main crest or prominent peaks and ridges. Cirques formed by cirque glaciers are generally restricted to the windward (west) side of the main crest. The few exceptions to this generalization are explained by similar circumstances on a more localized scale. The difference in snow accumulation and retention to which the two groups are attributable is the result of wind-drifted...
snow accumulation enhanced by orientations favorable to optimum shading on leeward slopes.

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REFERENCES