2016-17 AY Assessment Report
Masters of Arts in Mathematics
Department of Mathematics. Fresno State

1. **Learning outcomes assessed this year**

Direct Assessment.

We assessed one course this AY, MATH 251, which is the same one assessed last year. We are aware this is not ideal but our current committee was assembled in Spring 2017 and was unable to assess the Fall 2016 course as planned in our SOAP. This had the advantage of allowing the committee to study the SLO data for year-to-year variance.

The goals and SLOs assessed this year are:

Goal A. Provide students with advanced knowledge in the core areas of mathematics at the graduate level.

A1. Students will understand, describe, and illustrate the structural relationships among fundamental concepts in abstract algebra and real analysis (and geometry for students in the Teaching Option), such as function/transformation, derivative, integral, matrix, number/function set, algebraic structure (group, field, etc.).

Goal B. Continue development of students' ability to read, understand, and write rigorous mathematical proofs.

B2. Students will write advanced proofs in algebra and analysis (and geometry for students in the Teaching Option).

Goal C. Provide students with opportunities to apply mathematical knowledge to solve theoretical and practical problems.

C1. Students will utilize advanced problem-solving skills.

Indirect Assessment.

We looked at the online exit surveys our graduate committee collected from the 2016-17 graduating class. We also looked at the exit surveys generated in the last five years.

Other.

We compared our assessment of MATH 251 with the assessment given last year to the same course. This comparison is presented in item 3 below.

In addition, we looked even further back to see passing rates and grade distribution in MATH 251 in the last ten years.

2. **What instruments did you use to assess them?**

We embedded three questions in the final exam of MATH 251, and each of these questions was assessed for two distinct SLOs from A1, B2, and C1. Please find the questions assessed and their rubrics at the end of this document.

Exit surveys were used to capture students' feelings, and thoughts, about our program. We revised the surveys for the 2016-17 graduating class, and also looked at the surveys from the previous five years because (1) the data generated by these (older) surveys was never included in previous assessment reports, and (2) we get very few exit survey replies every year and so the data generated in a single year is pretty statistically irrelevant; hence looking at the last five years' surveys gives us a larger pool of data to look for patterns.

We used data available on Tableau to gain a historic perspective on the passing rates and grade distribution in MATH 251. We also used last year's assessment of this course for further comparison.

3. **What did you discover from these data?**

- All four students in the roster took the final exam. The course grade distribution was: A, B, B, and C. The final exam was take-home as was last year's; this is customary in several classes in our graduate program. One of the questions in the final was also used in the final last year; the instructor used the same grading rubric used last year.

  SLO A1. The instructor thinks that a score 9/12 (75%) in this question would be satisfactory. However, she expects a 10/12 (83%) average score.

  Students' scores were as follows: 8/12, 10/12, 10/12, 12/12 for an average of 10/12. Three of the four students performed satisfactorily, even to the higher expectations the instructor had, one of them did not.

  SLO B2. The instructor thinks that a score of 8/12 (67%) in this question would be satisfactory. However, she expects 9/10 or 10/12 (75% - 83%) to be the average score.

Remark: This is the problem that was used in last year's assessment.

Students' scores were as follows: 3/12, 5/12, 8/12, 12/12 for an average of 7/12. Two of the four students performed satisfactorily, other two did not.
Last year, the average score for this problem was 9/12 with score distribution 4/12, 4/12, 11/12, 11/12, 12/12, 12/12.

SLO C1. The instructor thinks that a score 12/16 (75%) in this question would be satisfactory. However, she expects a higher average score.

Students' scores are as follows: 11/16, 12/16, 13/16, 16/16, for an average of 13/16. Three of the four students performed satisfactorily, one of them did not.

Summarizing. The SLO's measured by embedded questions are generally met by the students. Three of four students achieved a satisfactory outcome (9 points out of 12) on A1. On outcome B2, two students achieved a satisfactory outcome (8 points out of 12) and two students did not. On outcome C1, three out of four students achieved a satisfactory outcome (12 points out of 16), though the last student was close. This is in line with the assessment of MATH 251 in the Spring 2016 semester. Five of six students satisfied A1 and A2 with one close; four of six students satisfied B2, with the instructor noting the two seemingly lacking effort; and five of six students satisfied C1.
We can conclude that students satisfy the A1 and C1 SLOs and struggle with SLO B2. This is not unexpected, as SLO's A1 and C1 measure more basic skills than B2. Moreover, SLO B2 requires students to develop strategies to write solutions and proofs; it needs them to put together a plan to apply important and complex theorems and identify intermediate steps or goals that may be useful toward a solution of the problem.

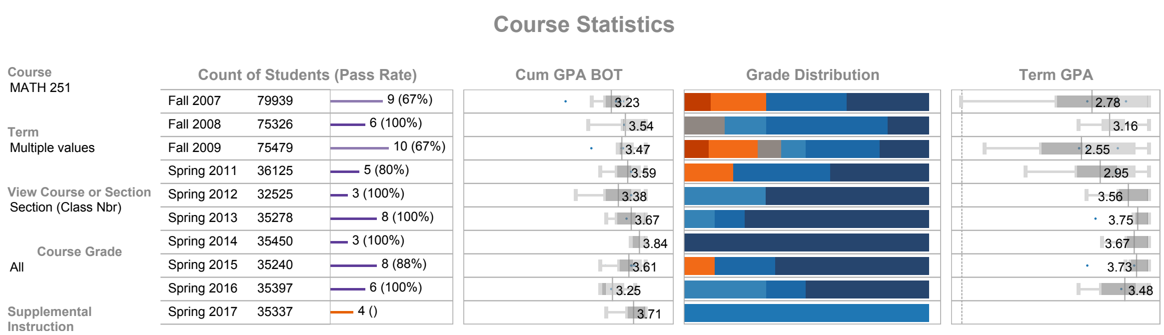
- The exit surveys (last five years) are fairly positive. Students feel their training was good and that they learned valuable skills while in our program. The places where we have some room for improvement, according to these surveys, are:
  (a) Our program focuses almost exclusively on the pure side of mathematics and misses on showing the value of mathematics in the world and how mathematics may be applied to 'real life' problems.
  (b) Our program should improve the quality and amount of information/advising about career paths that could possibly fit students' skills after getting their Masters; beyond the possibility of them teaching.
  An interesting piece of information is that students overwhelmingly answered they wanted to go into some kind of teaching to the question "What are your plans for next year?". Two students answered "to pursue a Ph.D. in mathematics".

There were no other types of answers.

(c) Mathematical software (LaTeX, GeoGebra, etc) does not play an important role in our program. However, LaTeX is required, for students in the pure option, at the time of typesetting a project/thesis (LaTeX is optional for students in the teaching option). Our program should do a better job at introducing, and requiring mathematical software knowledge from its students.

(d) Students who taught for us, as TA's, felt that they could have received better training to perform this job.

- In the course statistics dashboard on Tableau we can see that MATH 251 has a high passing rate, not so strange in graduate courses in general, but that also shows a healthy distribution of grades (given by the different shades of colors in the figure below: dark blue= A, lighter blue = B, even lighter blue = C, orange=D, red = F, and grey=I).



This seems to indicate that this class is at a good level for a Masters program in mathematics; students are able to succeed in it, with different degrees of achievement. We think this supports our department's selection of this class as a core class for all students in our Masters program.

4. **What changes did you make as a result of the findings?**

Our department does not currently have a plan for 'closing the loop'. Hence, actions will be taken after our department discusses our recommendations, and only if the faculty in our department agrees with our assessment and recommendations. Also, starting next year, the assessment of our graduate program will be done by our graduate committee; it will be this committee's job to close the loop on our recommendations.

After analyzing the data obtained, we identified two issues that we believe should be addressed:

(i) Students should be able to correctly apply big results/theorems while constructing proofs which are longer than just two steps. Multiple types of arguments (or elementary proof techniques) could/should be in use. They should also be able to devise plans for proving difficult theorems/results, i.e. they should be able to identify intermediate steps goals that may be useful toward the solution of the problem.

Regarding this issue, we make the following recommendations to our department.

- Each proof-based graduate course should require students to write complex proofs involving: (1) several steps and/or intermediate goals/lemmas, and (2) the devising of a plan that involves known results and the aforementioned intermediate goals. These problems could be given to students as short papers, or in-class assignments, take-home exams, etc.
- SLO B1 should be re-written using the language above.
- The graduate writing requirement should include, at least, (1) the statement of a major mathematical result and its proof (or a major mathematics education result and its background), and (2) the application of an argument/proof along the lines of the previous recommendations.

(ii) Our program should work on preparing its students better in applications of mathematics, appreciation of mathematics' place in the world, and also on mathematical software literacy.

There were no recommendations in our department's assessment report for the 2015-16 AY, and so there are no projected changes to be followed up in this report.

5. **What assessment activities will you be conducting in the 2017-18 academic year?**

(a) Embedded questions in MATH 271 (core course) and MATH 220 (recently created course).

(b) Create rubrics for evaluation of projects/theses and project/thesis defenses. These would also serve as report forms for thesis/project committees.

(c) Study the addition of familiarity with current technology accepted by the mathematical community to the Student Learning Outcomes. This could be done by modifying SLO C1 or by creating an entirely new SLO. The committee must make sure that the technological knowledge acquired by students in our Masters program should make them more competitive in the eyes of future employers.

(d) Create SOAP for future usage in the currently-being-developed MS program. This includes studying SLOs for new courses, courses that have not been taught in some time, etc.

6. **What progress have you made on items from your last program review action plan?**

   During the 2016-17 AY, we had a site visit to review our programs. The visit occurred on Sept. 28th and 29th, 2016. The review panel consisted of Prof. Kim Morin, Theatre Arts, CSU Fresno, Dr. Saeed Attar, Professor of Chemistry, Director of Honors College, CSU Fresno, and Dr. Ivona Grzegorczyk, Professor and Chair, Department of Mathematics, California State University, Channel Islands.

   The panel delivered the following recommendations. We have also included corrections to a few significant inaccurate statements in the report.

   A. Curriculum Improvements and Vision for the Future.

   Recommendation 1. The reviewers strongly support the proposal for a MS program in mathematics that includes common core mathematics courses and tracks in mathematics education, statistics, pure and applied mathematics. We urge the administration to help the department put the proposal on the 'fast track' for approvals.
   Response. The panel was confused about us already working on the process to convert our MA to an MS; we were not at the time. We have now formed a committee to study how to do the degree designation change. This process cannot be done 'fast-track' but the process is simple enough for us to be able to say that our MS program will be running in Fall 2019.

   Recommendation 2. Faculty should discuss a long-term vision for the graduate program in mathematics.
   Response. The degree designation change mentioned above will mean several changes to our program. These changes will show our vision for the future of our Masters program. Also, there will be lines open for future changes, such as the creation of a Statistics option, and the implementation of a 4+1 program.

   B. Graduate Student Enrollment

   Recommendation 1. Include Common Core courses and the applied and statistics track in the new M.S. proposal.
   Response. The MA-to-MS subcommittee is currently working on this. It has already been decided that there will be two core courses required for all students in the program. However, it is not clear whether other 'core' courses will be considered for each of the options. As of now, we cannot afford to offer more than four courses per semester.
   We have no plans to create Applied and Statistics Options at this time; however we have recently created a new applied course so we can give our students ways to get better applied training, if they wish to do so. Once our program is stronger (i.e. with more students), we

will revisit this issue.

Recommendation 2. Revisit the admission criteria, especially the required GRE scores.
Response. Our department discussed this during this AY and decided to continue requiring the math subject GRE score of 500 points. However, this requirement will be discussed again during Fall 2017, as our department chair and graduate coordinator believe that our GRE requirement is the main cause for the decreasing number of students in our program.

Recommendation (Administration) 3. Identify funding sources for tuition waivers.
Response. No progress.
On the other hand, mathematics *faculty* (not administration) are actively looking for sources of funding (no tuition waivers). During the 2017-18 AY, faculty will apply to the Bridge to Doctorate grant (with Physics and EES), and also to a Robert Noyce Scholarship Program (NSF grant as well).

C. Supporting Faculty Research and Workload Issues

Recommendation (Administration, Graduate School and the Department). Identify sources for long-term funding so the program can offer release time or summer stipends to faculty engaging in research and grant-writing activities.
Response. No progress.
On the other hand, two task forces addressing these issues have been created: A Provost task force on research and graduate studies, and a Senate task force on workload.

D. Graduate Program Budget

Recommendation (Administration and the Department). Identify College and University funds to be included in the departmental funding base for faculty scholarly activities and curriculum coordination.
Response. No progress.

E. Improving Technology Use in Mathematics Courses

Recommendation. Rethink delivery of graduate courses to include updated use of technology and current mathematical software.
Response. No progress.
No discussion has been had in this regard. However, individual faculty have been reacting to this recommendation and enhancing their use of technology in their teaching.
A related issue is listed, in item 5, as something we will address next year.

F. Supporting Graduate Student Research

Recommendation (Administration). Create funding for the department to support small courses for faculty student research projects.
Response. No progress.
On the other hand, our department's former Undergraduate Research Committee has been 'expanded' to include graduate students as well. Hence, the funding opportunities that were available only to undergraduates are now also available for graduate students. This does not precisely generate funds for small courses but it will, in principle, address/promote faculty-student research at the graduate level.

G. Facilities

Recommendation (Administration) 1. Try to place all faculty and graduate student in offices that are in closer proximity to the department.
Response. Our Dean has provided three additional offices for our part time faculty to share. Even after this, our need for part time faculty and TA office space remains severely limited.

Recommendation (Administration) 2. Provide additional space that is equipped appropriately for best practices in teaching mathematics that will facilitate faculty/student collaboration and research activities.
Response. No progress.

H. Assessment and Student Learning Outcomes

Recommendation. The Student Learning Outcomes should include familiarity with current technology accepted by the mathematical community.
Response. There are two SLOs that touch on this. They are:
C2. Students will enhance computational and visualization skills by utilizing mathematical software.
D2. Students will be able to use technology in written reports and oral presentations.

Since these SLOs have never been assessed, we have added this issue to the assessment plan for next year (see item 5).

MATH 251, Assessment Rubric. Spring 2017

<u>SLO A1.</u> Let R be an (additive) Abelian group and let

$$End(R) = \{\phi : R \to R \mid \phi \text{ is a group homomorphism}\}$$

(a) Prove that $End(R)$ is a ring with one under the usual sum and composition of functions.
(b) Prove that the units of $End(R)$ are the automorphisms of R.
(c) Assume R is a simple finite group. Prove that $End(R)$ is a division ring.

Solution. Total of 12 points

- **1 point**. Addition of homomorphisms is a homomorphism; addition in $End(R)$ is commutative;
- **1 point**. Addition in $End(R)$ is associative;
- **1 point**. The zero element in $End(R)$; argue that the zero function is a group homomorphism;
- **1 point**. Additive inverse in $End(R)$; argue that $-\phi$ is a homomorphism;
- **1 point**. Composition of functions in $End(R)$ is associative; composition of homomorphisms is a homomorphism;
- **1 point**. Let and right distributivity of composition over addition of functions;
- **1 point**. Unity in $End(R)$; argue that the identity function on R is a homomorphism;
- **2 points**. Prove that ϕ is a unit if and only if it is bijective. They are allowed to use without proof that a function is bijective if and only if it has an inverse.
- **1 point**. Know the definition of simple group and that $\ker(\phi)$ is a normal subgroup of R.
- **1 point**. Conclude that if $\phi \neq 0$, then ϕ must be one-to-one (as otherwise, $\ker(\phi)$ is a proper normal subgroup of R.)
- **1 point**. Using that if R is finite and $\phi \neq 0$ is injective, then ϕ is onto, and thus bijective. Conclude that every $\phi \neq 0$ is a unit and thus $End(R)$ is a division ring.

<u>SLO B2.</u> If N is a finite normal subgroup of G and $P \in Syl_p(N)$, prove that $G = N \cdot N_G(P)$. In particular, prove that if P is normal in N, then P is normal in G.

Solution. Total of 12 points

- **1 point**. We note first that $N \cdot N_G(P) \subset G$ (since $N \subset G$, $N_G(P) \subset G$ and G is closed under composition).
- **2 points**. Let $g \in G$. Since $P \subset N$ and N is normal in G, then $gPg^{-1} \subset N$. Moreover, $gPg^{-1} \leq N$ and $|gPg^{-1}| = |P|$. Thus, $gPg^{-1} \in Syl_p(N)$.
- **2 points**. Then, by Sylow's Theorem for the group N, there exists $n \in N$ such that $gPg^{-1} = nPn^{-1}$.
- **2 points**. Then, $n^{-1}gPg^{-1}n = n^{-1}nPn^{-1}n = P \Longrightarrow (n^{-1}g)P(n^{-1}g)^{-1} = P$.
- **2 points**. It follows that $n^{-1}g \in N_G(P)$, which implies that $g \in nN_G(P)$. Since g was arbitrarily chosen, we have shown that $G \subset N \cdot N_G(P)$, and therefore, $G = N \cdot N_G(P)$.
- **3 points**. Suppose now that P is normal in N. Then $N \subset N_G(P)$, so

$$N \cdot N_G(P) = N_G(P) \Longrightarrow G = N_G(P) \Longrightarrow P \text{ is normal in } G.$$

SLO C1. Classify all groups of order 20, up to isomorphism (there are five isomorphism types).

Solution. Total of 16 points

- **2 points**. $|G| = 2^2 \cdot 5$. If G is Abelian, then we know by the Fundamental Theorem of Finite Abelian Groups that there are two isomorphism types: $\mathbb{Z}_{20}$ and $\mathbb{Z}_{10} \times \mathbb{Z}_2$.
- **1 point**. Knowing that $n_5(G) = 1$, $|Syl_5(G)| = 1$, and thus $P \lhd G$, where $Syl_5(G) = \{P\}$. Noting that $P \cong \mathbb{Z}_5$.
- **1 point**. Knowing that $Syl_2(G) \neq \emptyset$ and that $n_2(G) \in \{1, 5\}$.
- **1 point**. Arguing that $P \cap Q = \{e_G\}$, where $Q \in Syl_2(G)$ (by Lagrange's Theorem, since $|P| = 5$ and $|Q| = 4$ and gcd$(4, 5) = 1$).
- **1 point**. Arguing that $PQ \leq G$. Using that $|PQ| = |P||Q|$ to conclude that $PQ = G$ and recognize that $G \cong P \rtimes_\phi Q$, for some homomorphism $\phi : Q \to Aut(P)$.
- **1 point**. Knowing that classifying the non-Abelian groups of order 20 is equivalent to determining the nonisomorphic groups constructed as a semidirect product, using homomorphisms ϕ that are non-trivial maps.
- **1 point**. Knowing that since $|Q| = 4$, then $Q \cong \mathbb{Z}_4$ or $Q \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.
- **1 point**. Knowing that $Aut(P) \cong Aut(\mathbb{Z}_5) \cong \mathbb{Z}_4$. Let $Aut(P) =< \alpha >$. Then $|\alpha| = 4$. For example $\alpha(y) = y^2$, where $P =< y >$.

Case 1. $Q \cong \mathbb{Z}_4$ (cyclic). Let $Q =< x >$. Thus $|x| = 4$.

- **1 point**. Knowing that there are three non-trivial homomorphisms $\phi : Q \to Aut(P)$. Noting that the trivial homomorphism $\phi_1(x) = 1$ yields

$$P \rtimes_{\phi_1} Q \cong P \times Q \cong \mathbb{Z}_5 \times \mathbb{Z}_4 \cong \mathbb{Z}_{20}$$

- **1 point**. Writing correctly the three non-trivial homomorphisms:

$$\phi_2(x) = \alpha,\ \ \phi_3(x) = \alpha^2,\ \ \phi_4(x) = \alpha^3$$

- **1 point**. Arguing that $P \rtimes_{\phi_4} Q \cong P \rtimes_{\phi_2} Q$, since $\phi_4 \circ \beta = \phi_2$, where $\beta : Q \to Q$ is the automorphism of Q given by $\beta(x) = x^{-1} = x^3$.

- **1 point**. Arguing that $P \rtimes_{\phi_3} Q \not\cong P \rtimes_{\phi_2} Q$, since $ker(\phi_3) \not\cong ker(\phi_2)$. (Note $ker(\phi_3) = \{1, x^2\} \cong \mathbb{Z}_2$ and $ker(\phi_2) = \{1\}$.) Thus there are two non-Abelian groups (up to isomorphism) of order 20 with a cyclic Sylow 2-subgroup: $\mathbb{Z}_5 \rtimes_{\phi_2} \mathbb{Z}_4$ and $\mathbb{Z}_5 \rtimes_{\phi_3} \mathbb{Z}_4$.

Case 2. $Q \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ (Elementary Abelian). Let $Q =< a > \times < b >$. Thus $|a| = 2 = |b|$.

- **1 point**. Then any homomorphism $\psi : Q \to Aut(P)$ satisfies $|\psi(a)|, |\psi(b)| \in \{1, 2\}$. (The trivial homomorphism $\psi_1(a) = \psi_1(b) = 1$ yields $P \rtimes_{\psi_1} \cong \mathbb{Z}_{10} \times \mathbb{Z}_2$, which is Abelian.)

- **1 point**. Students need to know that there are three non-trivial homomorphisms ψ:

$$\begin{array}{ll} \psi_2(a) = \alpha^2 & \psi_2(b) = 1 \\ \psi_3(a) = 1 & \psi_3(b) = \alpha^2 \\ \psi_4(a) = \alpha^2 & \psi_4(b) = \alpha^2 \end{array}$$

- **1 point**. Argue that

$$\mathbb{Z}_5 \rtimes_{\psi_3} (\mathbb{Z}_2 \times \mathbb{Z}_2) \cong \mathbb{Z}_5 \rtimes_{\psi_2} (\mathbb{Z}_2 \times \mathbb{Z}_2)$$

(via $\theta(a) = b, \theta(b) = a$ where $\theta \in Aut(\mathbb{Z}_2 \times \mathbb{Z}_2)$) and that

$$\mathbb{Z}_5 \rtimes_{\psi_4} (\mathbb{Z}_2 \times \mathbb{Z}_2) \cong \mathbb{Z}_5 \rtimes_{\psi_2} (\mathbb{Z}_2 \times \mathbb{Z}_2)$$

(via $\alpha(a) = a, \alpha(b) = ab$ where $\alpha \in Aut(\mathbb{Z}_2 \times \mathbb{Z}_2)$). Thus, there is a unique non-Abelian group (up to isomorphism) of order 20 with an Elementary Abelian Sylow 2-subgroup: $\mathbb{Z}_5 \rtimes_{\psi_2} (\mathbb{Z}_2 \times \mathbb{Z}_2)$.